

A stochastic integer programming approach to the optimal thermal and wind generator scheduling problem

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York University*

*Industrial Optimization Seminar
Fields Institute for Research in Mathematical Science*

March 6th, 2012

Outline

Overview



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Problem statement



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Stochastic programming model



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Computational challenge and solution



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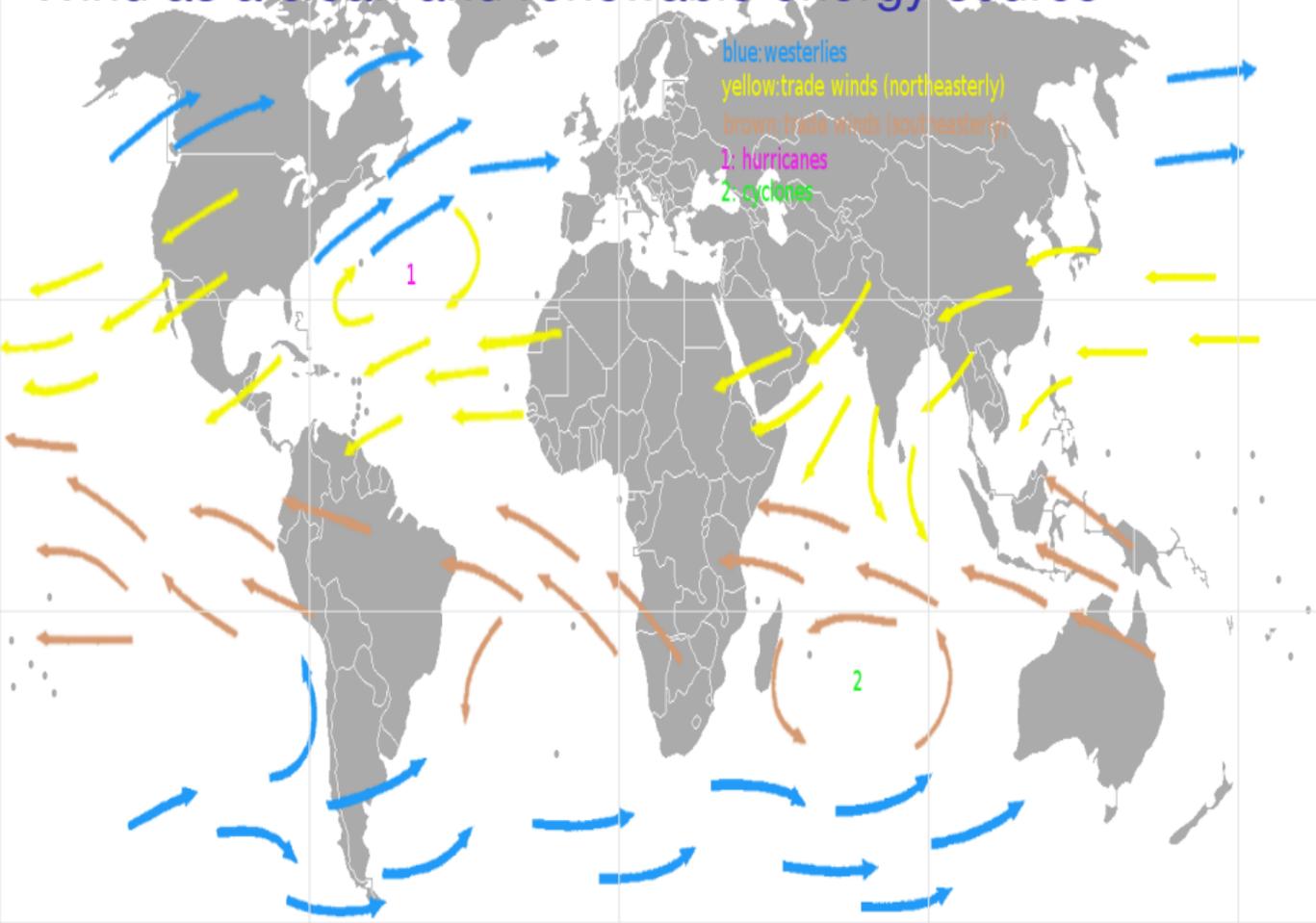
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Benchmarking report



Wind as a clean and renewable energy source



Canada's current
installed capacity:

5,265 MW



Current status of Ontario wind energy

ONTARIO WIND GENERATORS



Can you balance?



demand = generation?

Can you balance?



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stochastic demand = generation?

Can you balance?

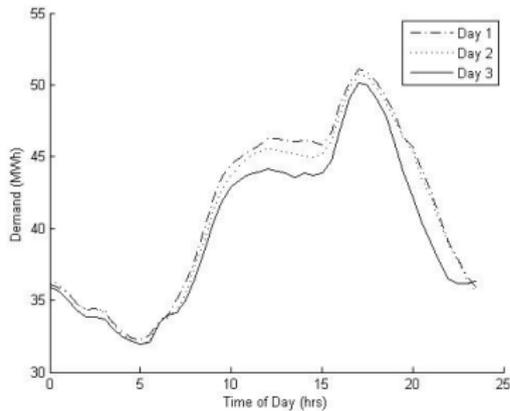


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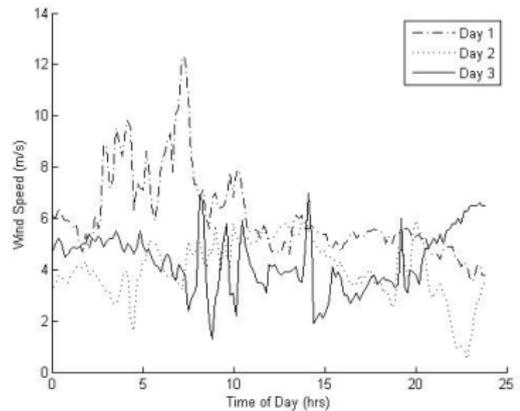
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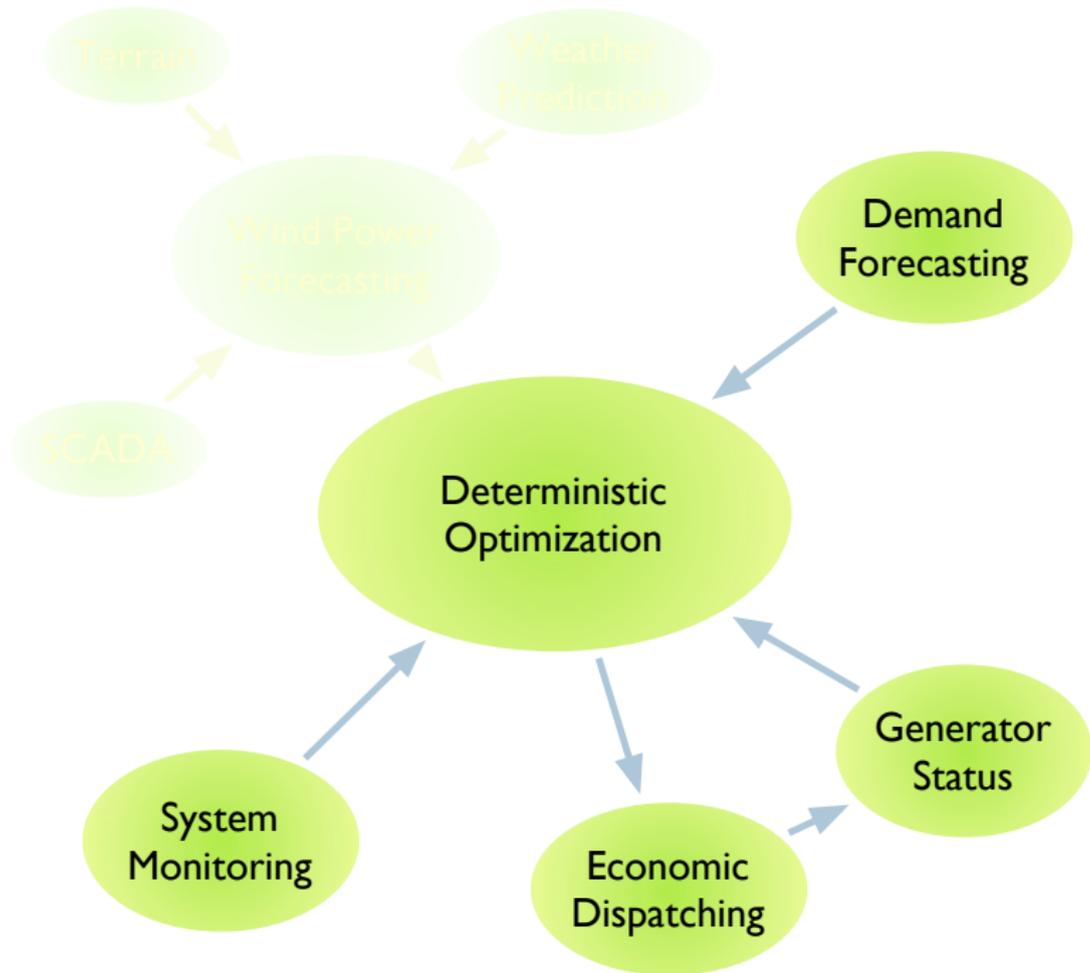
Increasing wind and increasing system volatility



(a) Daily Demand Curves



(b) Daily Wind Speed Curves



All we need is prediction?



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*Prediction is very difficult,
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– Niels Bohr*



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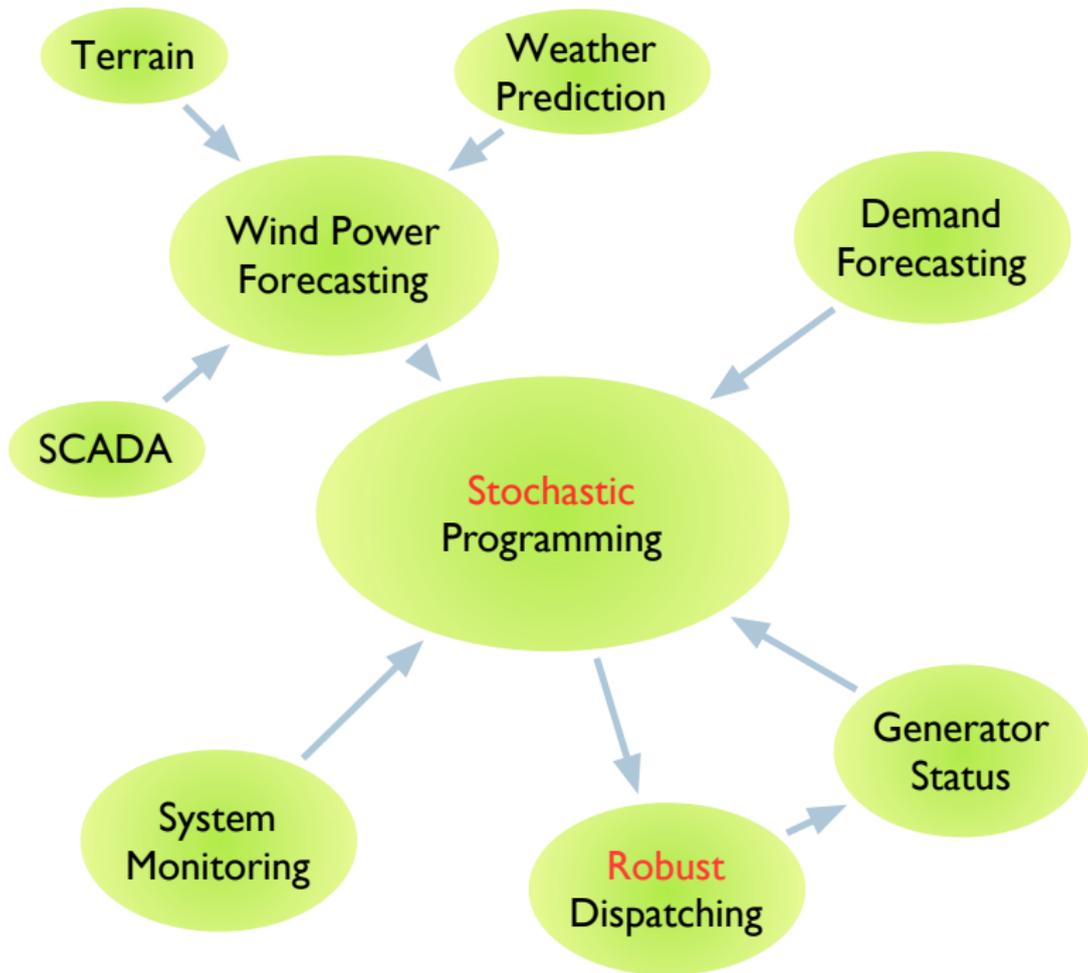
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- ▶ *we need operational flexibility less expensive than reserve.*



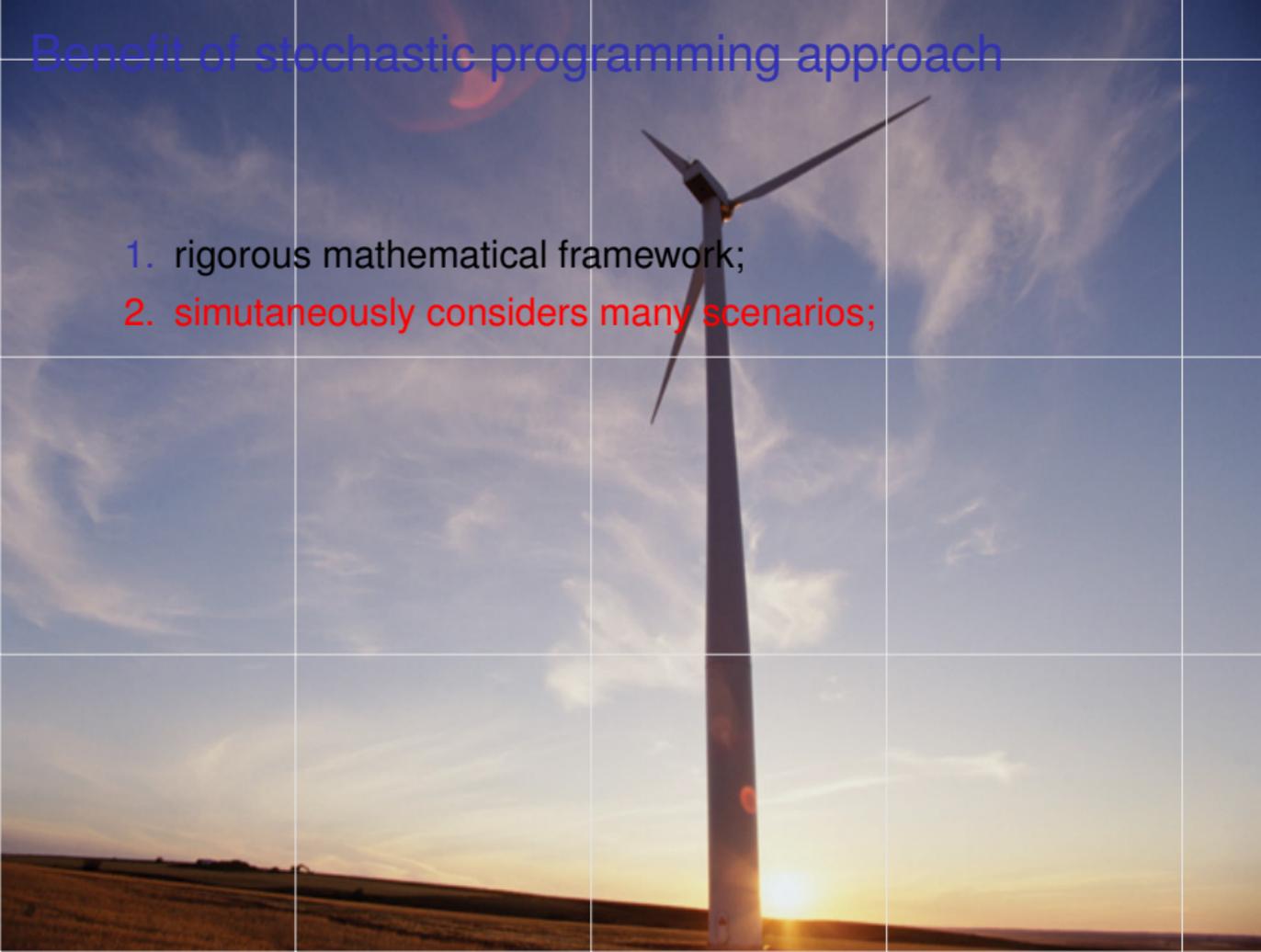
Benefit of stochastic programming approach

1. rigorous mathematical framework;

A tall, white wind turbine stands against a dramatic sunset sky. The sun is low on the horizon, creating a bright orange glow and casting long, wispy clouds. The sky is a mix of blue and orange. A white grid is overlaid on the entire image.

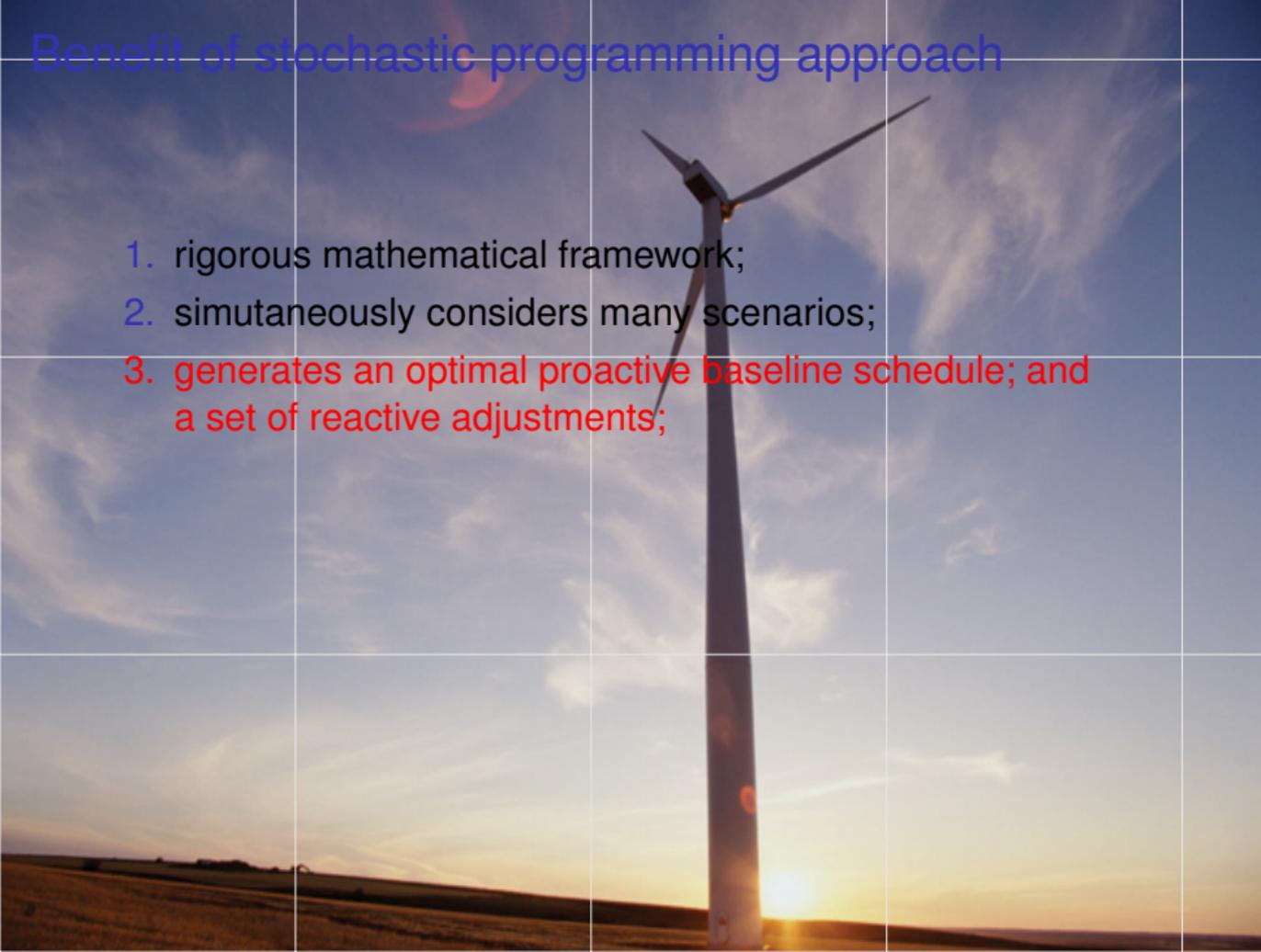
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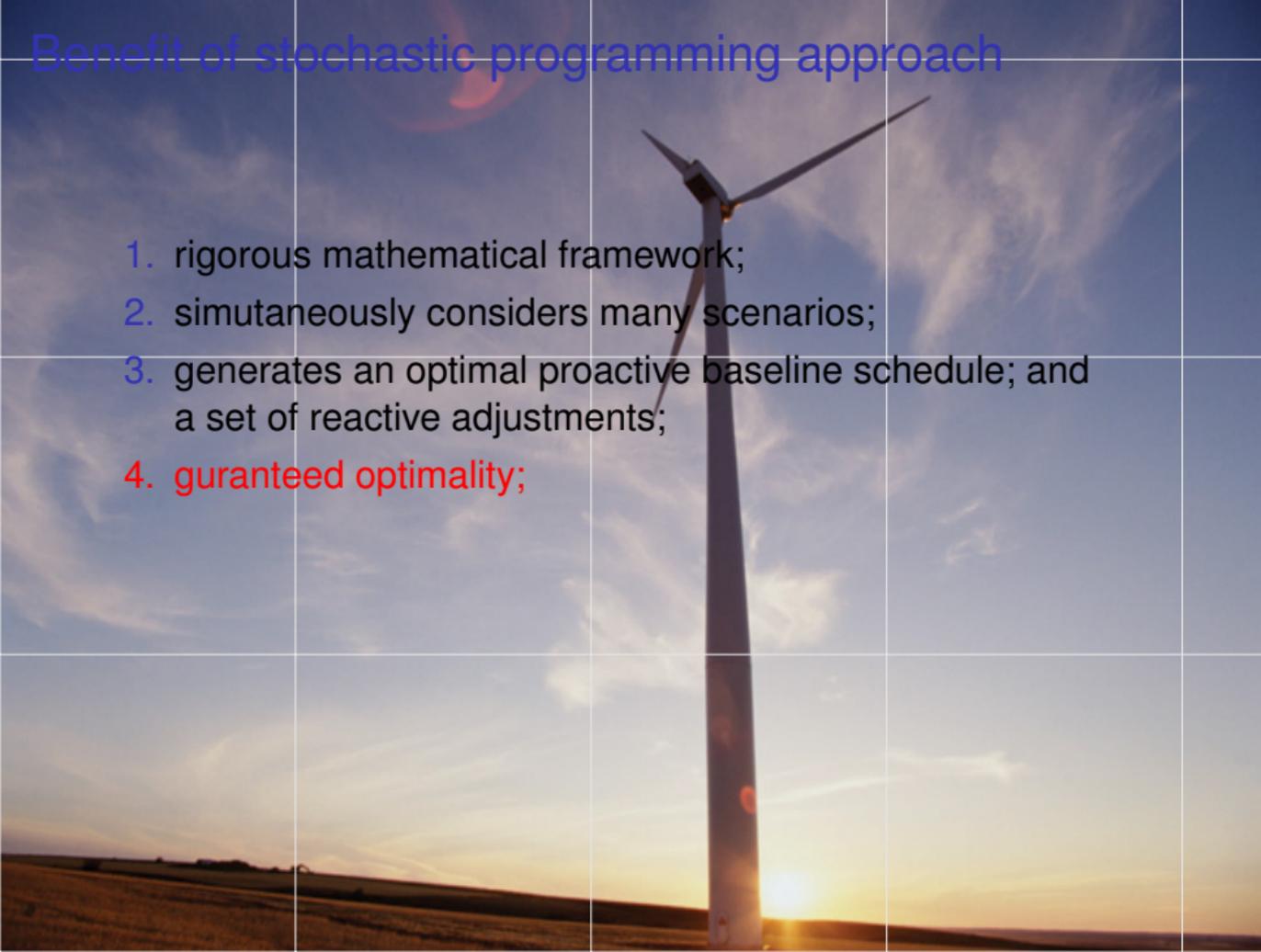
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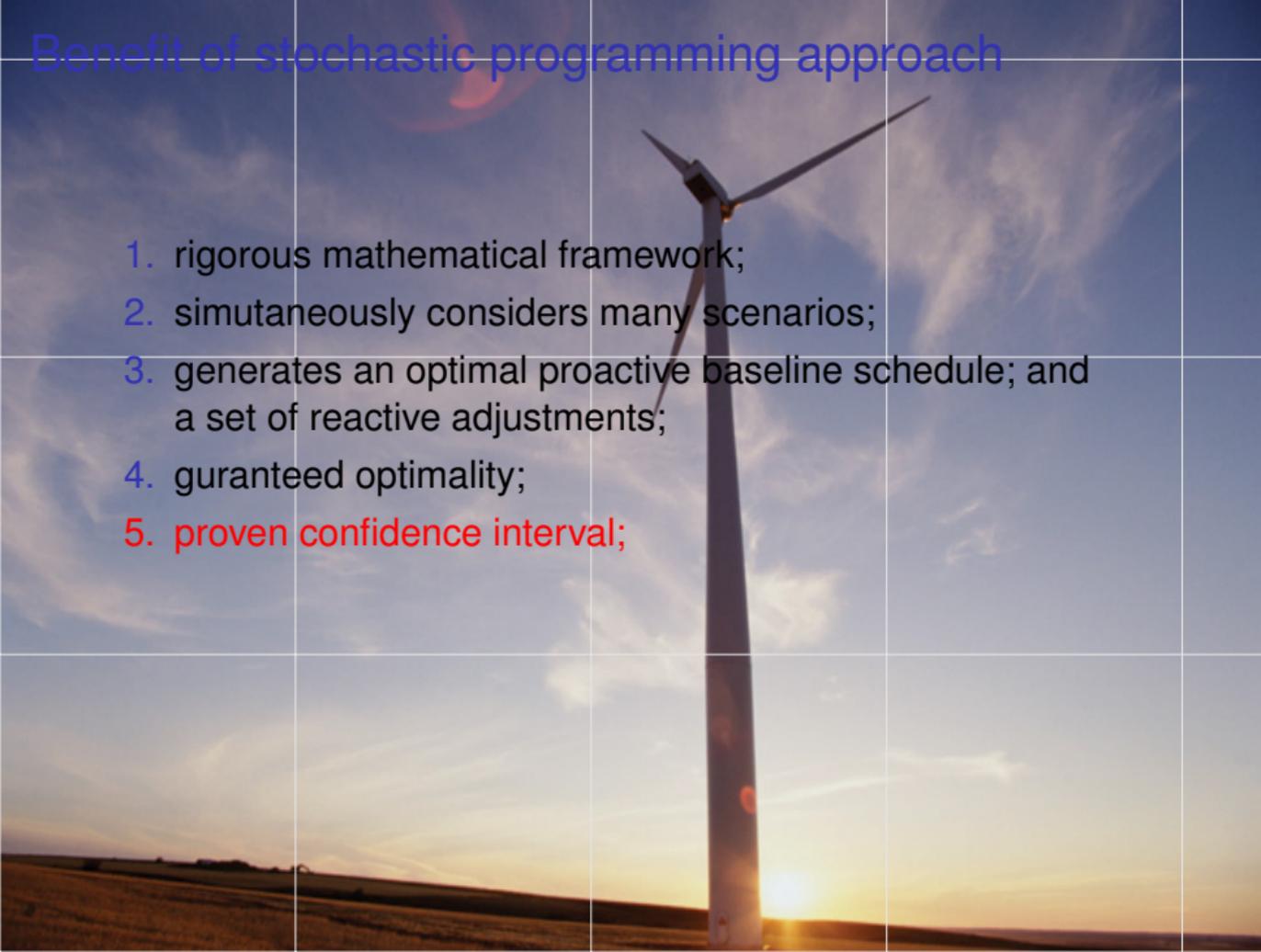
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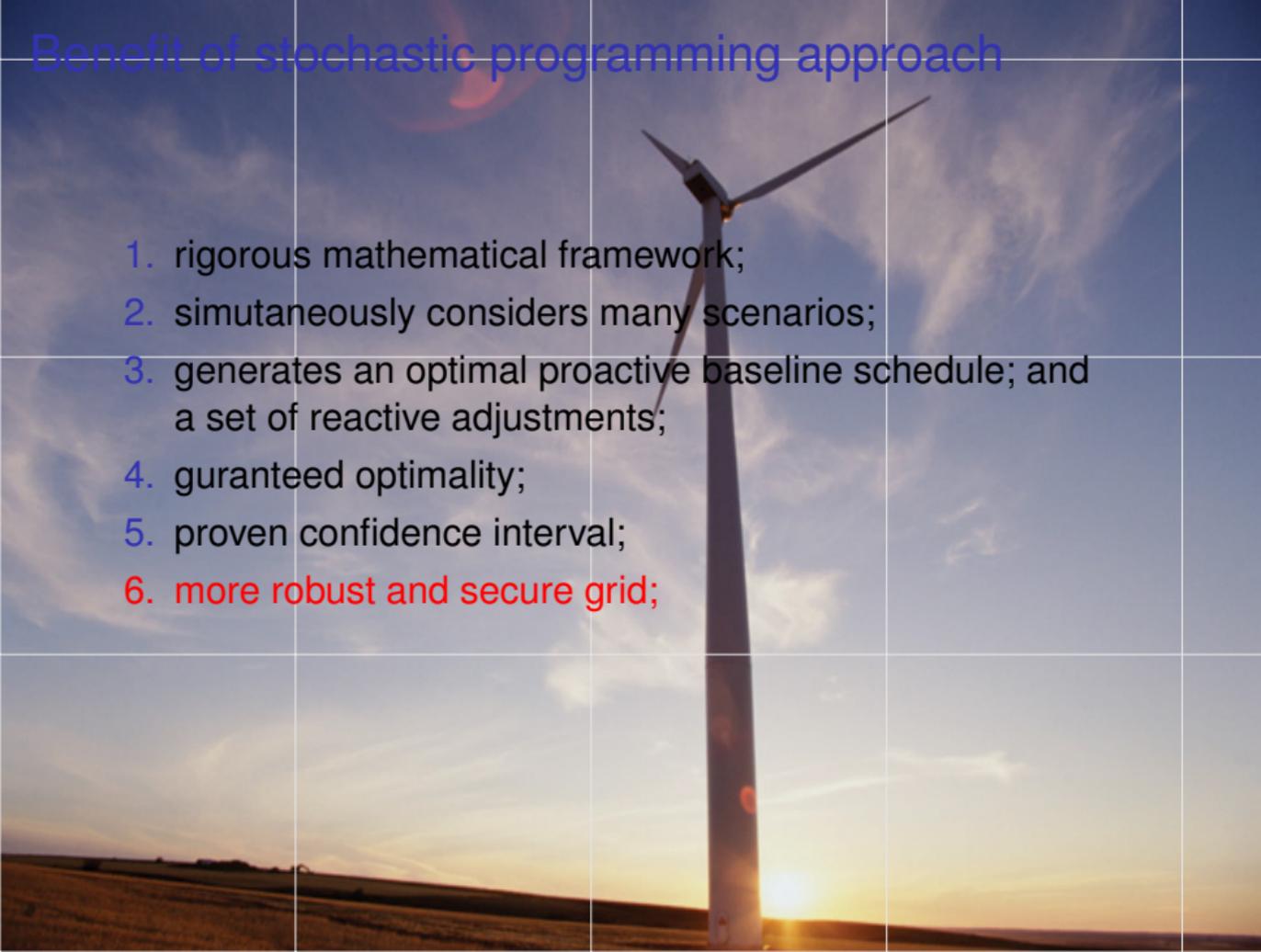
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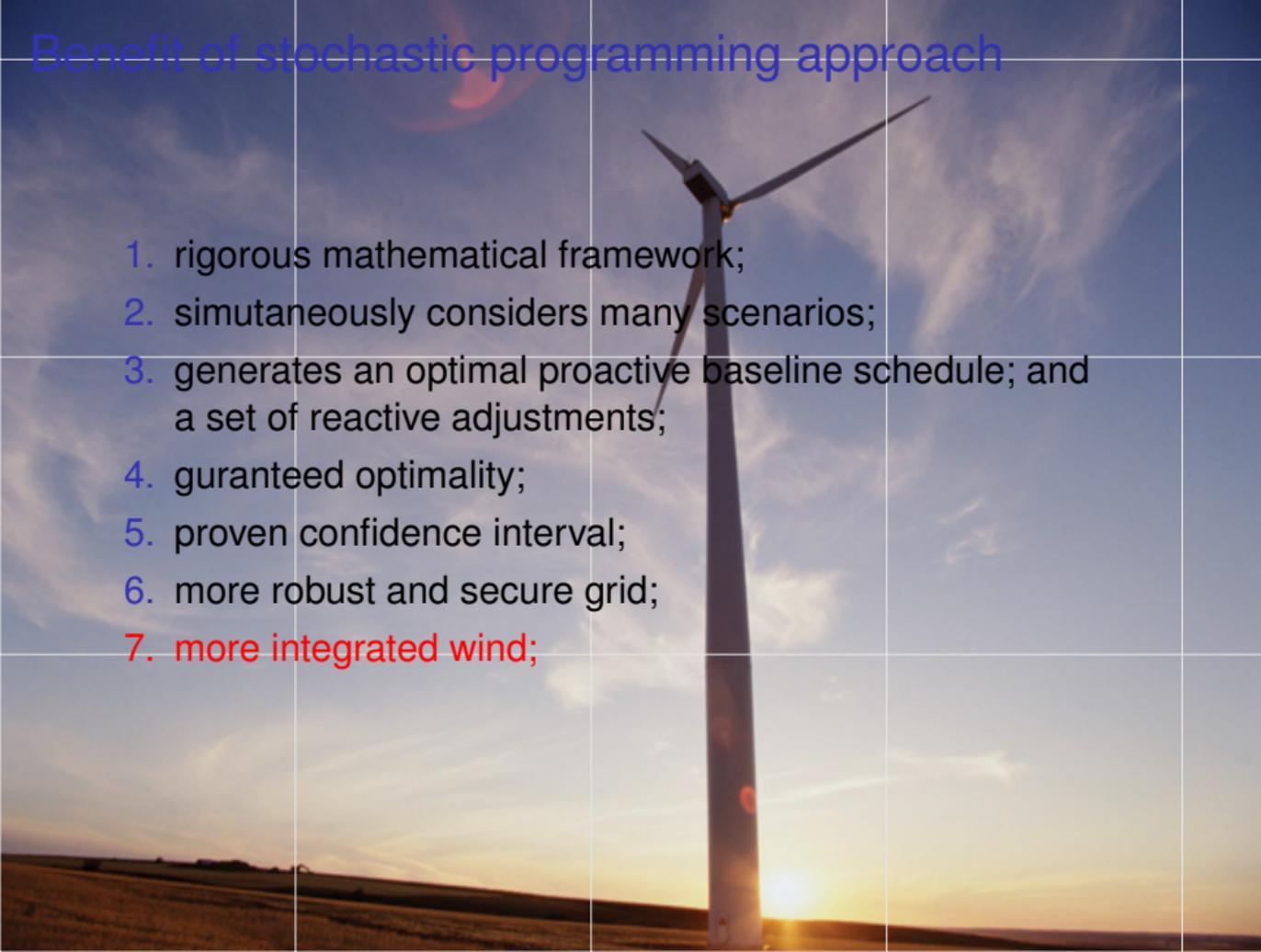
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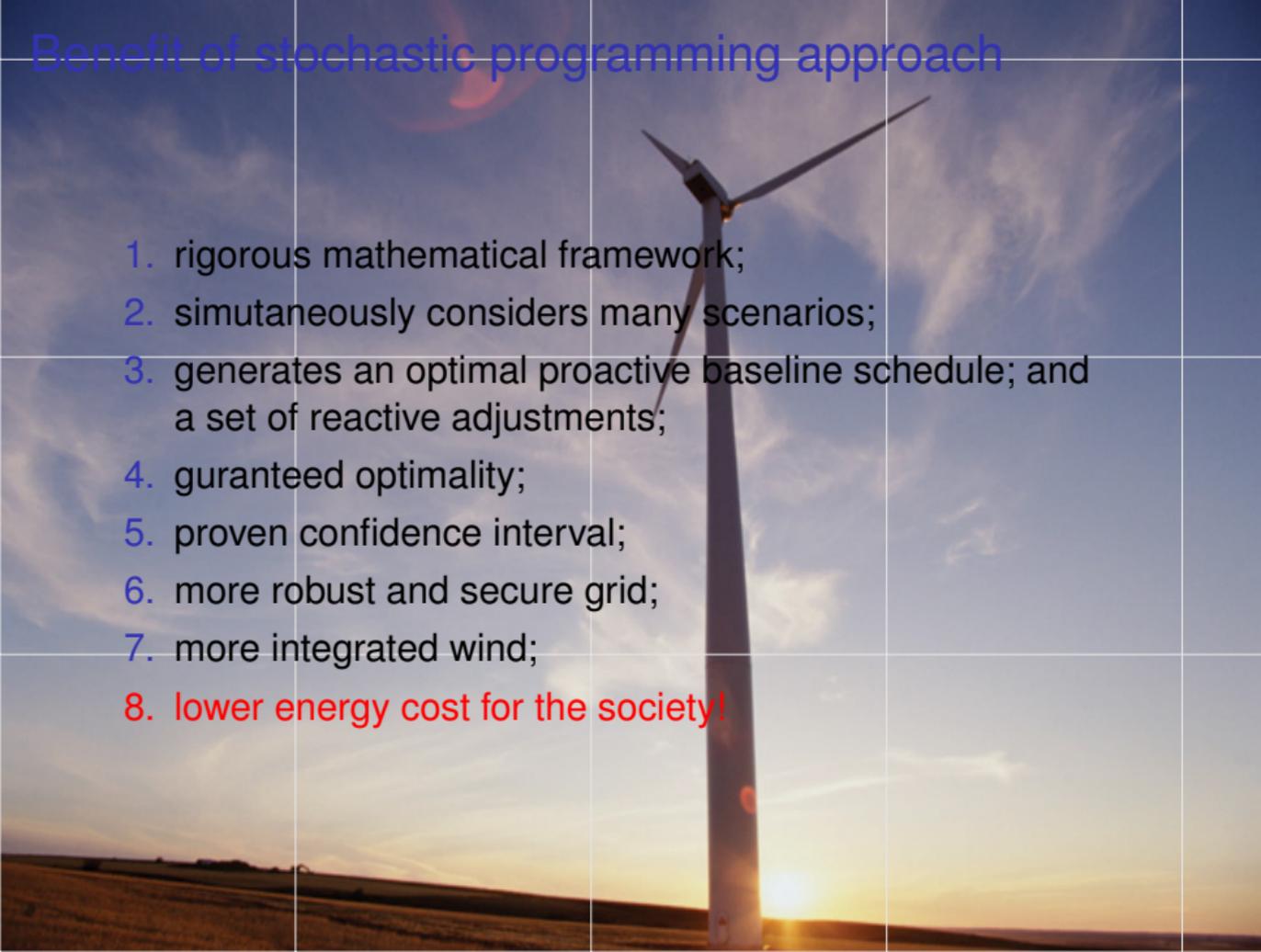
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6. more robust and secure grid;
7. more integrated wind;
8. lower energy cost for the society!



Stochastic programming model: key decision variables

v_{ih} : on/off status of slow generator i at each time
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Stochastic programming model: key decision variables

v_{ih} : on/off status of slow generator i at each time h , $\forall i \in J_s$;

v_{ih}^s : on/off status of fast generator i at each time h in scenario s , $\forall i \in J_f, s \in S$;

All other variables are dependent on the variable v .

Stochastic programming model

$$\min \overbrace{\sum_{\substack{h \in H \\ j \in J_s}} (c_j^u y_{jh} + c_j^d z_{jh} + c_j^m v_{jh})}^{\text{first stage}} + \sum_{s \in \mathcal{S}} \pi^s \overbrace{\left(\sum_{\substack{h \in H \\ j \in J_f}} (c_j^u y_{jh}^s + c_j^d z_{jh}^s + c_j^m v_{jh}^s) + \sum_{\substack{h \in H \\ j \in J}} c_j^p p_{jh}^s \right)}^{\text{second stage}}$$

$$s.t. (\mathbf{y}_{J_s}, \mathbf{z}_{J_s}, \mathbf{v}_{J_s}) \in \mathcal{U}_{J_s} \quad (1)$$

$$(\mathbf{y}_{J_f}, \mathbf{z}_{J_f}, \mathbf{v}_{J_f}) \in \mathcal{U}_{J_f}^s, \forall s \in \mathcal{S} \quad (2)$$

$$(\mathbf{p}^s, \mathbf{w}^s) \in \mathcal{C}^s, \forall s \in \mathcal{S} \quad (3)$$

$$(\mathbf{p}^s, \mathbf{w}^s) \in \mathcal{N}^s, \forall s \in \mathcal{S} \quad (4)$$

$$(\mathbf{p}^s, \mathbf{w}^s) \in \mathcal{R}^s, \forall s \in \mathcal{S} \quad (5)$$

Stochastic programming model: unit commitment constraints

The unit commitment constraints for a generator $j \in J$ is defined as:

$$\mathcal{U}_j := \begin{cases} y_{jh} \geq v_{jh} - v_{j,h-1} & \forall h \in H & (6a) \\ z_{jh} \geq v_{j,h-1} - v_{jh} & \forall h \in H & (6b) \\ y_{j,h-\bar{T}_j+1} + \dots + y_{jh} \leq v_{jh} & \forall h \in H & (6c) \\ z_{j,h-\underline{T}_j+1} + \dots + z_{jh} \leq 1 - v_{jh} & \forall h \in H & (6d) \\ v_{j,h-1} - v_{jh} + y_{jh} - z_{jh} = 0 & \forall h \in H & (6e) \\ v_{jh} \in \{0, 1\}, y_{jh}, z_{jh} \in [0, 1] & \forall h \in H & (6f) \end{cases}$$

The constraint sets $\mathcal{U}_J, \mathcal{U}_{J_s}, \mathcal{U}_{J_f}$ are defined accordingly:

$$\mathcal{U}_J := \bigcap_{j \in J} \mathcal{U}_j, \mathcal{U}_{J_s} := \bigcap_{j \in J_s} \mathcal{U}_j, \mathcal{U}_{J_f} := \bigcap_{j \in J_f} \mathcal{U}_j \quad (7)$$

Stochastic programming model: reserve constraints

$$\mathcal{R}^s := \left\{ \begin{array}{ll} \sum_{j \in J} rs_{jh}^s \geq \eta_s \sum_{l \in L} d_{lh}^s & \forall h \in H \quad (8a) \\ \sum_{j \in J} rs_{jh}^s + \sum_{j \in J} ro_{jh}^s \geq \eta \sum_{l \in L} d_{lh}^s & \forall h \in H \quad (8b) \end{array} \right.$$

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- ▶ We assume the contingency constraint is implemented exogenously following NERC N-1 rule.

Stochastic programming model: network constraints

$$\mathcal{N}^s := \left\{ \begin{array}{ll} \sum_{j \in J_m} p_{jh}^s + \sum_{i \in I_m} w_{ih}^s + \sum_{nm \in E} f_{nmh}^s = & \\ \sum_{mn \in E} f_{mnh}^s + \sum_{l \in L_m} d_{lh}^s + gsh_m & \forall m \in V, h \in H \quad (9a) \\ f_{mnh}^s = b_{mn}(\theta_{mh}^s - \theta_{nh}^s - \gamma_{mnh}^s) & \forall mn \in E, h \in H \quad (9b) \\ -\bar{f}_{mn} \leq f_{mnh}^s \leq \bar{f}_{mn} & \forall mn \in E, h \in H \quad (9c) \\ \underline{\gamma}_{mn} \leq \gamma_{mnh}^s \leq \bar{\gamma}_{mn} & \forall mn \in E, h \in H \quad (9d) \\ \theta_{ref,h}^s = 0 & \forall h \in H \quad (9e) \end{array} \right.$$

Stochastic programming model: capacity constraints

$$c^s := \begin{cases} \underline{P}_j v_{jh} \leq P_{jh}^s & \forall j \in J, h \in H \quad (10a) \\ P_{jh}^s + rs_{jh}^s \leq \bar{P}_j v_{jh} & \forall j \in J, h \in H \quad (10b) \\ P_{jh}^s + rs_{jh}^s + ro_{jh}^s \leq \bar{P}_j & \forall j \in J, h \in H \quad (10c) \\ P_{j,h-1}^s - P_{j,h}^s \leq \underline{R}_j & \forall j \in J, h \in H \quad (10d) \\ P_{jh}^s - P_{j,h-1}^s + rs_{jh}^s + ro_{jh}^s \leq \bar{R}_j & \forall j \in J, h \in H \quad (10e) \\ rs_{jh}^s \leq \overline{RS}_j & \forall j \in J, h \in H \quad (10f) \\ ro_{jh}^s \leq \overline{RO}_j & \forall j \in J, h \in H \quad (10g) \\ w_{lh}^s \leq \tilde{w}_{lh}^s & \forall l \in L, h \in H \quad (10h) \end{cases}$$

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- ▶ both the first stage and the second stage of the stochastic unit commitment model are integer problem!
- ▶ cutting-plane method is very effective for integer problem;
- ▶ **how about stochastic integer problem?**

Our solution: scenario crossing deep cuts

Definition

$C_{sh} \subset J$ is a (s, h) -cover if

$$\sum_{j \in C_{sh}} \bar{P}_j + \sum_{i \in I} \tilde{w}_{ih}^s < (1 + \eta_s) \sum_{l \in L} d_{lh}^s.$$

If in addition

$$\sum_{j \in C_{sh}} \bar{P}_j + \sum_{i \in I} \tilde{w}_{ih}^s + \underline{P}_i \geq (1 + \eta_s) \sum_{l \in L} d_{lh}^s \quad \forall i \in J - C_{sh}, \quad (11)$$

then the cover C is simple.

Our solution: scenario crossing deep cuts

PROPOSITION

If $(1 + \eta_s) \sum_{l \in L} d_{lh_1}^{s_1} - \sum_{i \in I} \tilde{w}_{ih_1}^{s_1} \leq (1 + \eta_s) \sum_{l \in L} d_{lh_2}^{s_2} - \sum_{i \in I} \tilde{w}_{ih_2}^{s_2}$, then (i) a (s_1, h_1) -cover is also a (s_2, h_2) -cover; (ii) any (s_2, h_2) -cover has a (s_1, h_1) -cover subset.

Our solution: scenario crossing deep cuts

PROPOSITION

Let C be a (s, h) -cover and $\Delta_h^s = \sum_{l \in L} d_{lh}^s - \sum_{j \in C} \bar{P}_j - \sum_{i \in I} \tilde{w}_{ih}^s$.
Then the strengthened (s, h) -cut

$$\sum_{j \in J-C} \frac{p_{jh}^s}{\max\{\underline{P}_j, \Delta_h^s\}} + \sum_{i \in I} \frac{w_{ih}^s}{\Delta_h^s} \geq 1 \quad (12)$$

is valid for $\rho^s(\cdot)$. If in addition,

$$\Delta_h^s \leq \bar{P}_j, \forall j \in J - C,$$

and (11) holds strictly for some indices, then (12) is facet-defining for $\rho^s(\cdot)$.

Our solution: scenario crossing deep cuts

Definition

Two generators a and b are symmetric if a and b have identical physical features and are located on one bus.

PROPOSITION

Assume there are κ symmetric pairs in the electricity grid. Let Ω^s be the feasible set of $(\mathbf{v}^s, \mathbf{y}^s, \mathbf{z}^s)$ in $\rho^s(\lambda^s)$, and $\Omega^{s'}$ be the reduced feasible set after applying the κ symmetry cuts:

$$y_{ah}^s + v_{a,h-1}^s + v_{b,h-1}^s \geq y_{bh}^s, \text{ for all symmetric pairs } (a,b),$$

then

$$|\Omega^{s'}| = \frac{|\Omega^s|}{2^\kappa},$$

i.e., the feasible region shrinks exponentially.

Numerical test: RTS-96 system

Table: Generator Mix

Type	Technology	No. units	Capacity(MW)	list of units
U12	Oil/Steam	5	60	16-20
U20	Oil/CT	4	80	1-2,5-6
U50	Hydro	6	300	25-30
U76	Coal/Steam	4	304	3-4, 7-8
U100	Oil/Steam	3	300	9-11
U155	Coal/Steam	4	620	21-22, 31-32
U197	Oil/Steam	3	591	12-14
U350	Coal/3 Steam	1	350	33
U400	Nuclear	2	800	23-24
W150	Wind	1	150	.
W100	Wind	1	100	.

Numerical test: RTS-96 system

Table: Bus Generator Incidence

Bus	Generators	Bus	Generator	Bus	Generator
1	1-4	7	9-11	18	23
2	5-8	13	12-14	21	24
4	W150	15	16-21	22	25-30
5	W100	16	22	23	31-33

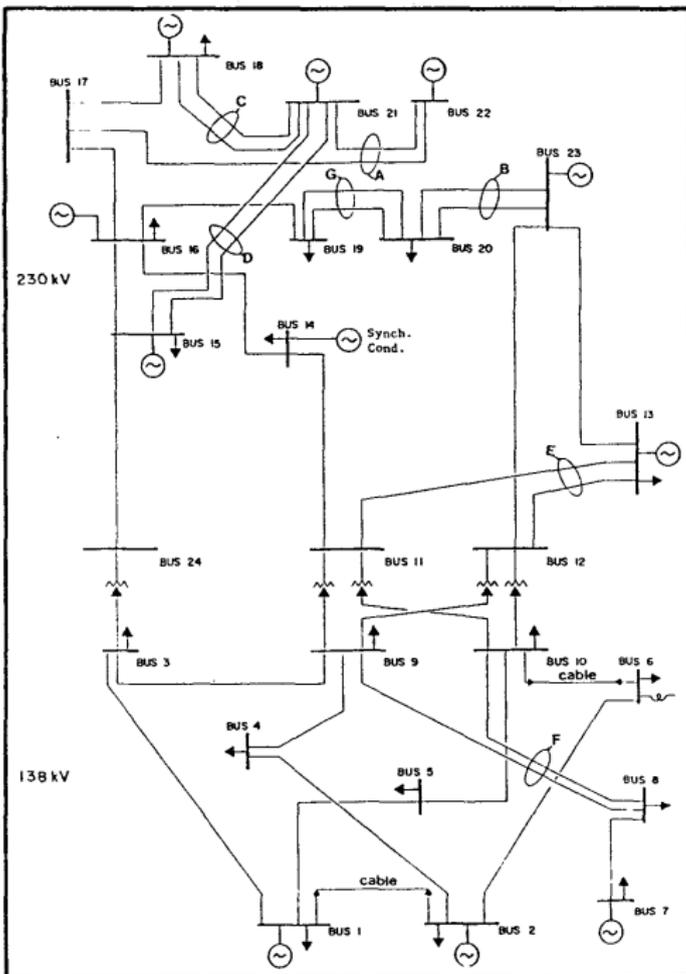


Figure 1 - IEEE One Area RTS-96

- ▶ 30 generators;
- ▶ 24 buses;
- ▶ 34 lines;
- ▶ installed capacity 3,405MW.

Effect of symmetry cut

Optimal schedule 1			Optimal schedule 2		
Gen	switch on	switch off	Gen	switch on	switch off
7	6	23	8	6	23
3	7	23	3	7	23
4	7	23	4	7	23
8	7	23	7	7	23
13	7	21	13	7	21
12	8	22	12	8	22
14	8	22	14	8	22
17	8	12	16	8	12
20	8	12	18	8	12
2	10	11	1	10	11

Table: Optimal Schedule for Two-scenario Instance. Generators not shown in the table are always on. No wind curtailment nor demand shedding appears in this optimal schedule. The optimal objective value is \$798,256. Total demands for the two scenarios over the 24 hrs are 56811.5MW and 56571.8MW.

Benchmarking report

S	CPLEX B&B		CPLEX D&S		Cover Cuts			Both cuts	
	nodes	seconds	nodes	seconds	nodes	seconds	Ncuts	node	second
2	3027	78	3453	75	2460	53	2	48	37
5	1051	73	2544	224	185	17	10	25	10
10	3715	703	3137	870	1741	529	12	117	101
15	37040	5053	44838	21500	1631	1064	23	1103	1176
20	5279	5023	8684	4096	3592	3359	29	237	282
25	7281	4191	23533	18998	1667	2920	42	621	1305

Table: Statistics of running time. The number of cuts shown in table is for cover cuts; the number of symmetry cuts is constantly 36, which is not shown in the table.

S	2	5	10	15	20	25	mean	median
cover cut	29%	76%	24%	78%	17%	30%	42%	29%
both cuts	50%	86%	85%	76%	93%	68%	76%	80%

Table: Reduction rate of running time

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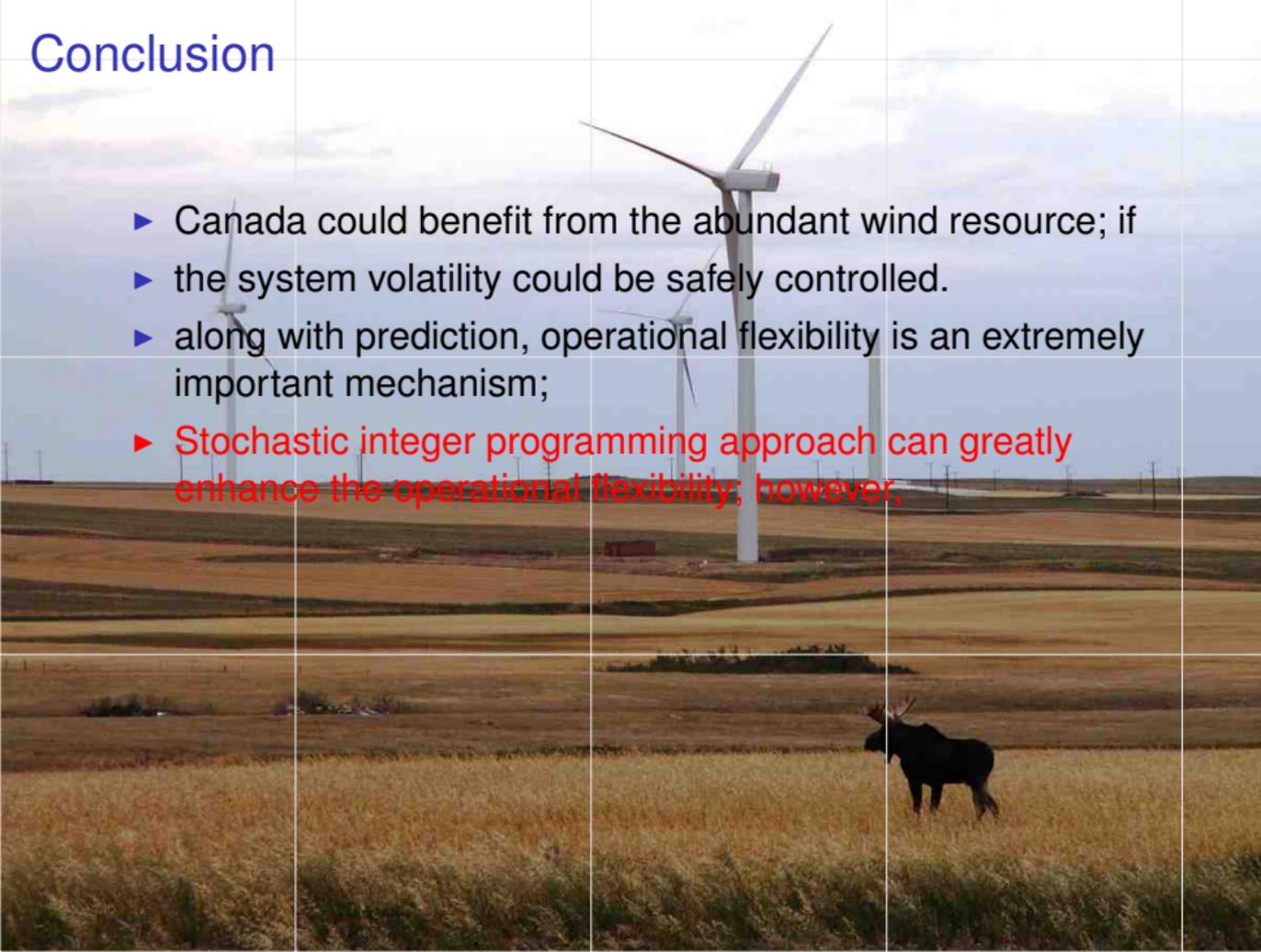
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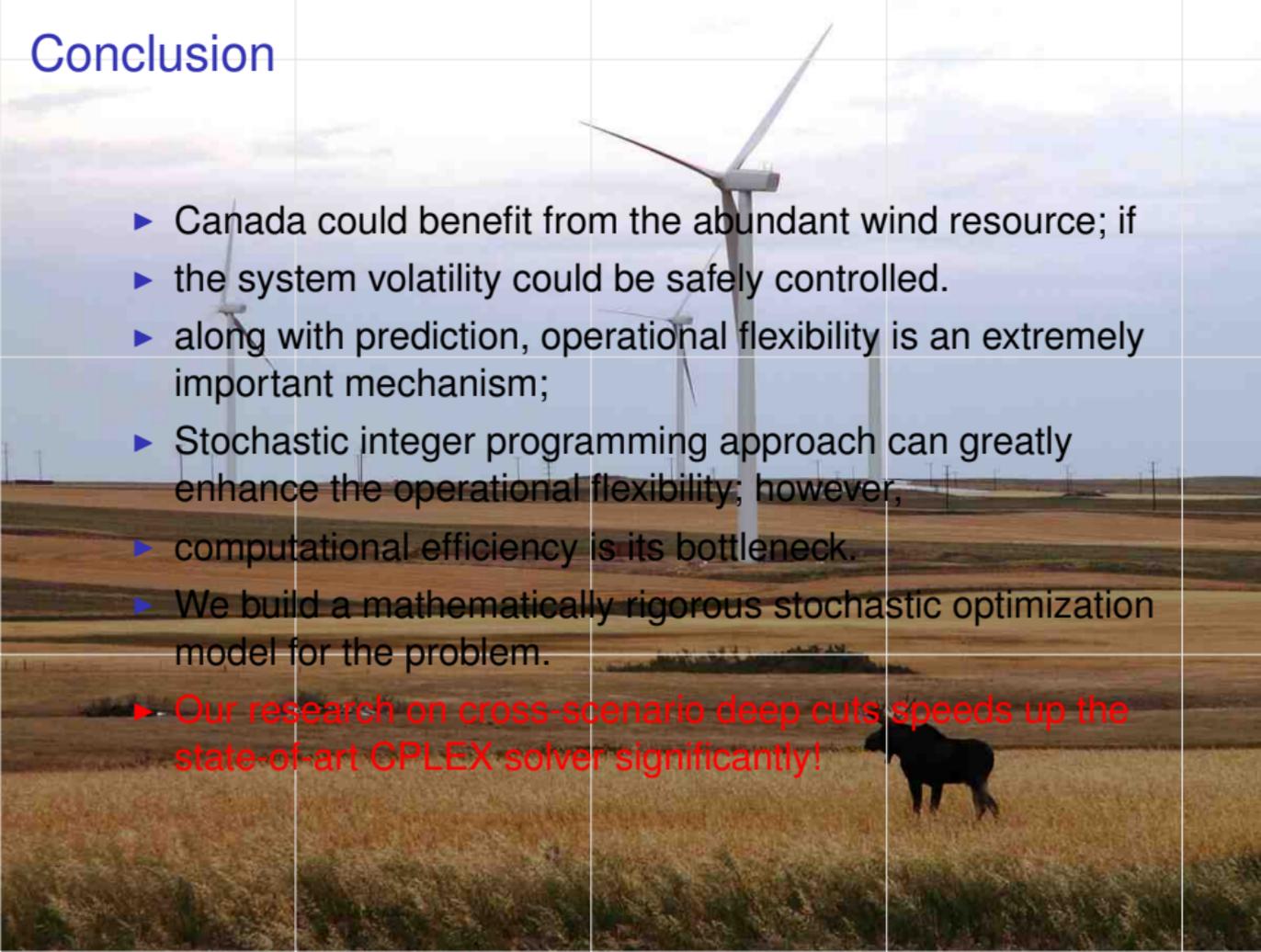
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Conclusion

The background of the slide is a photograph of a wind farm. Several white wind turbines are visible against a cloudy sky. The foreground shows a field of tall, golden-brown grass. In the lower right corner, a black cow is standing in the field. The overall scene is a rural landscape.

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- ▶ the system volatility could be safely controlled.
- ▶ along with prediction, operational flexibility is an extremely important mechanism;
- ▶ Stochastic integer programming approach can greatly enhance the operational flexibility; however,
- ▶ computational efficiency is its bottleneck.
- ▶ We build a mathematically rigorous stochastic optimization model for the problem.
- ▶ Our research on cross-scenario deep cuts speeds up the state-of-art CPLEX solver significantly!

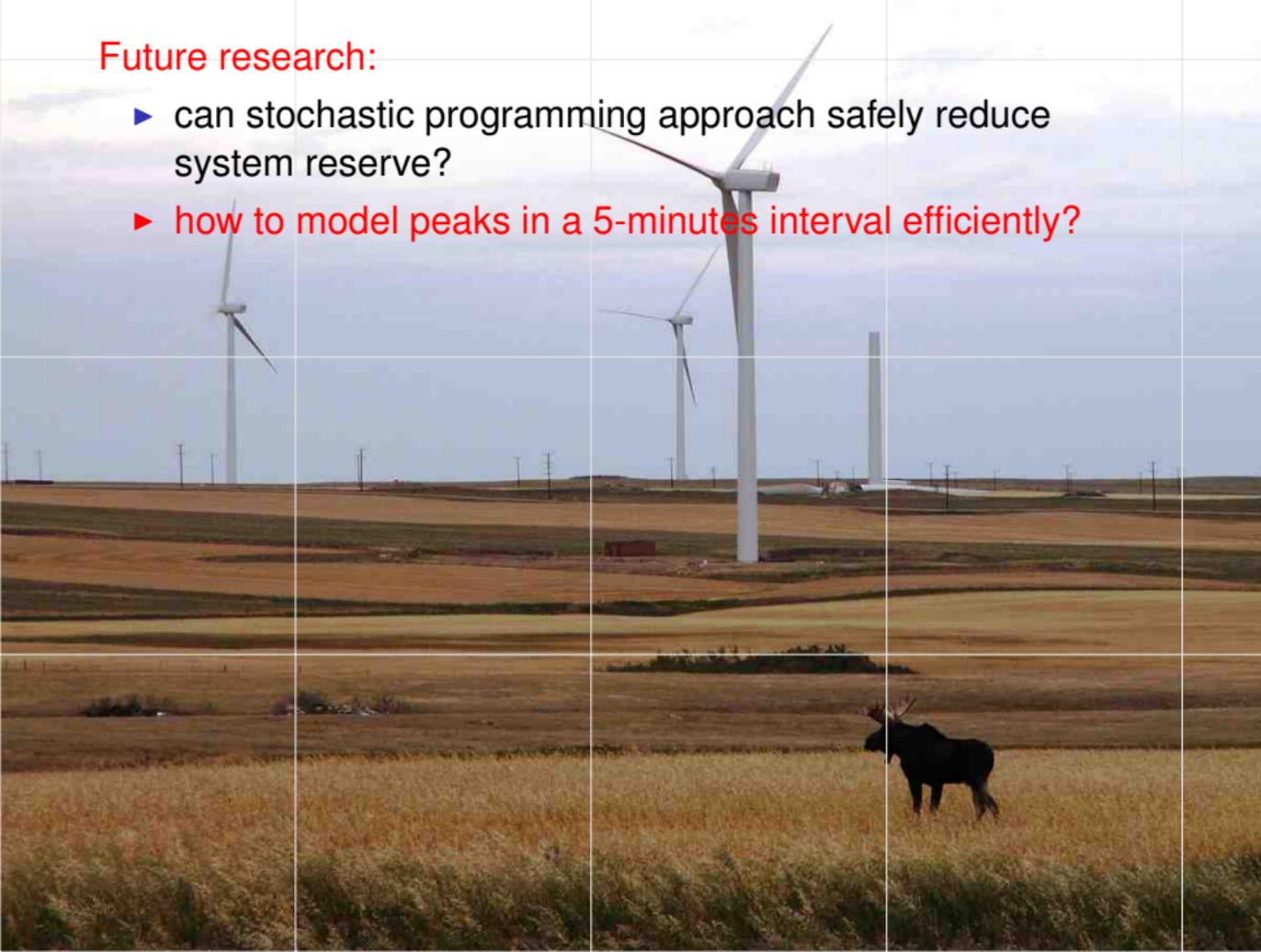
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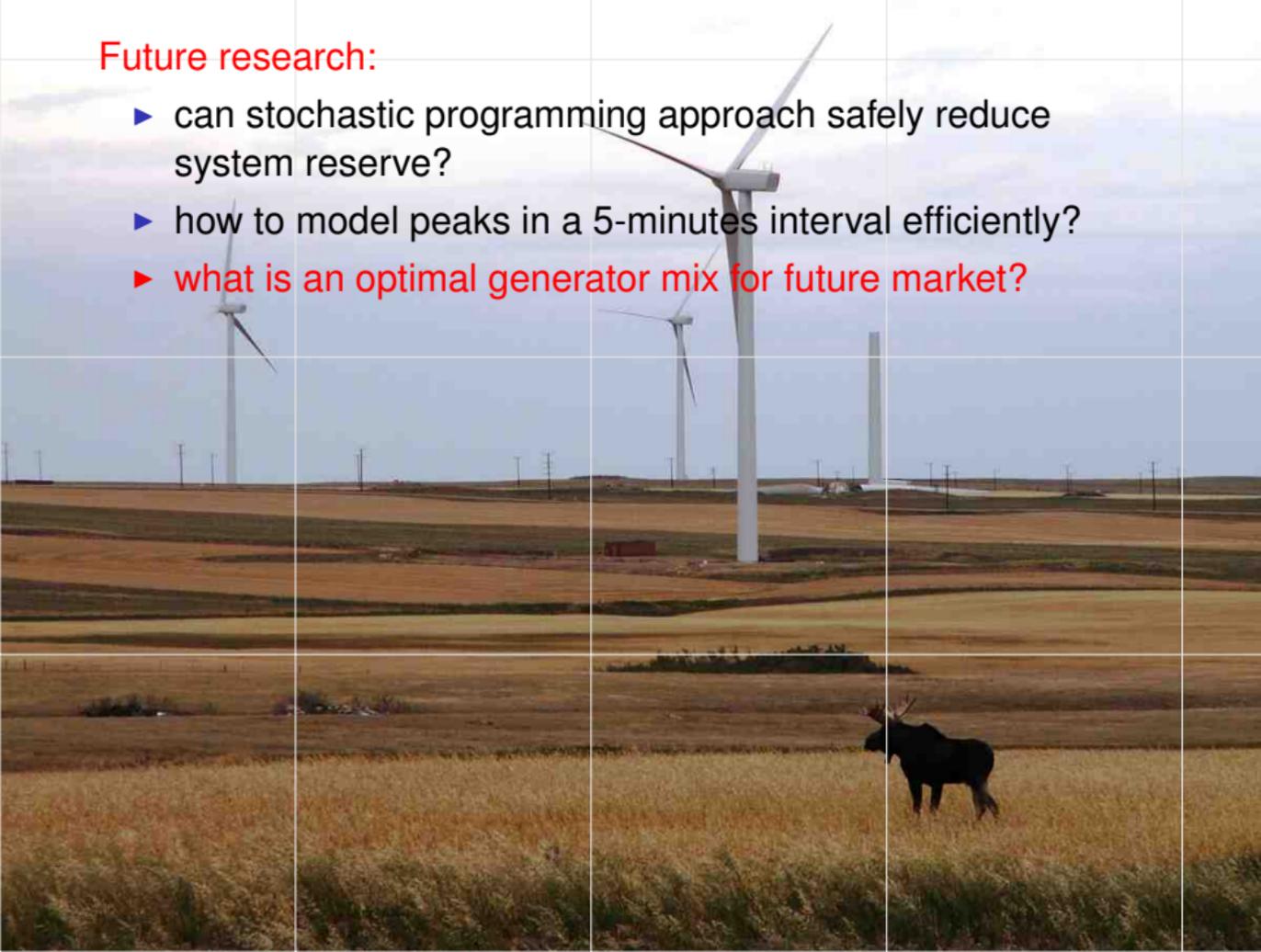
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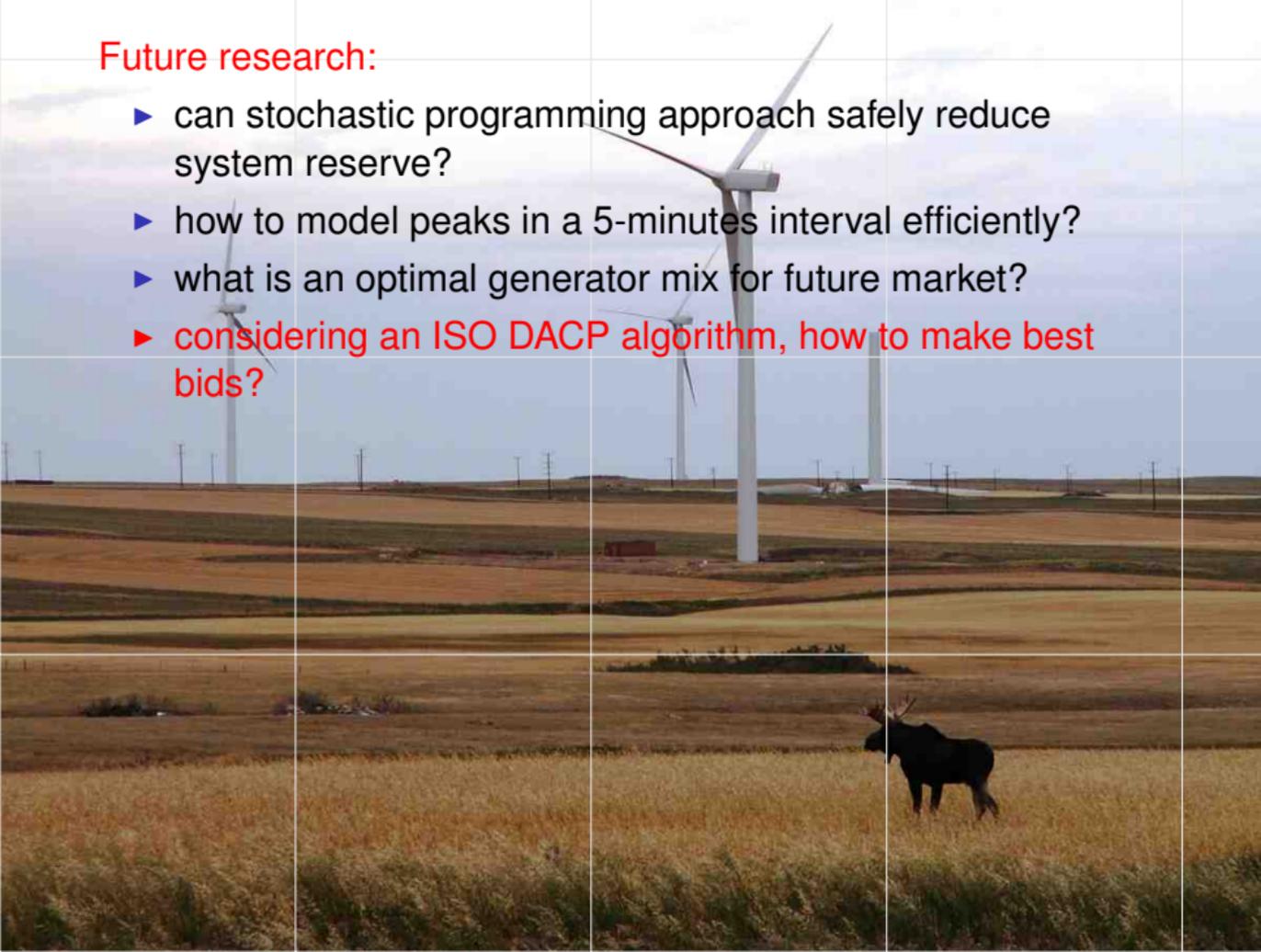
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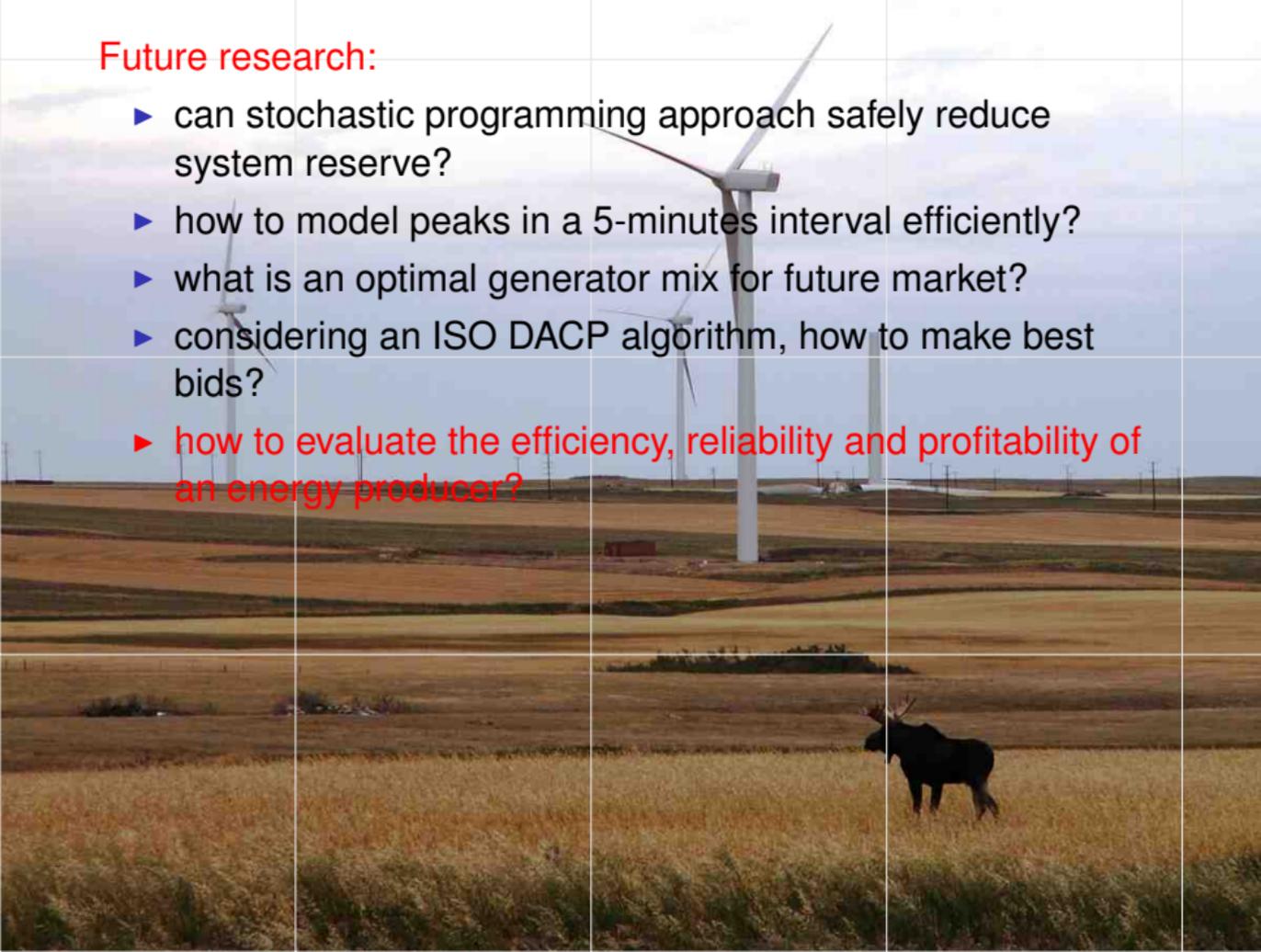
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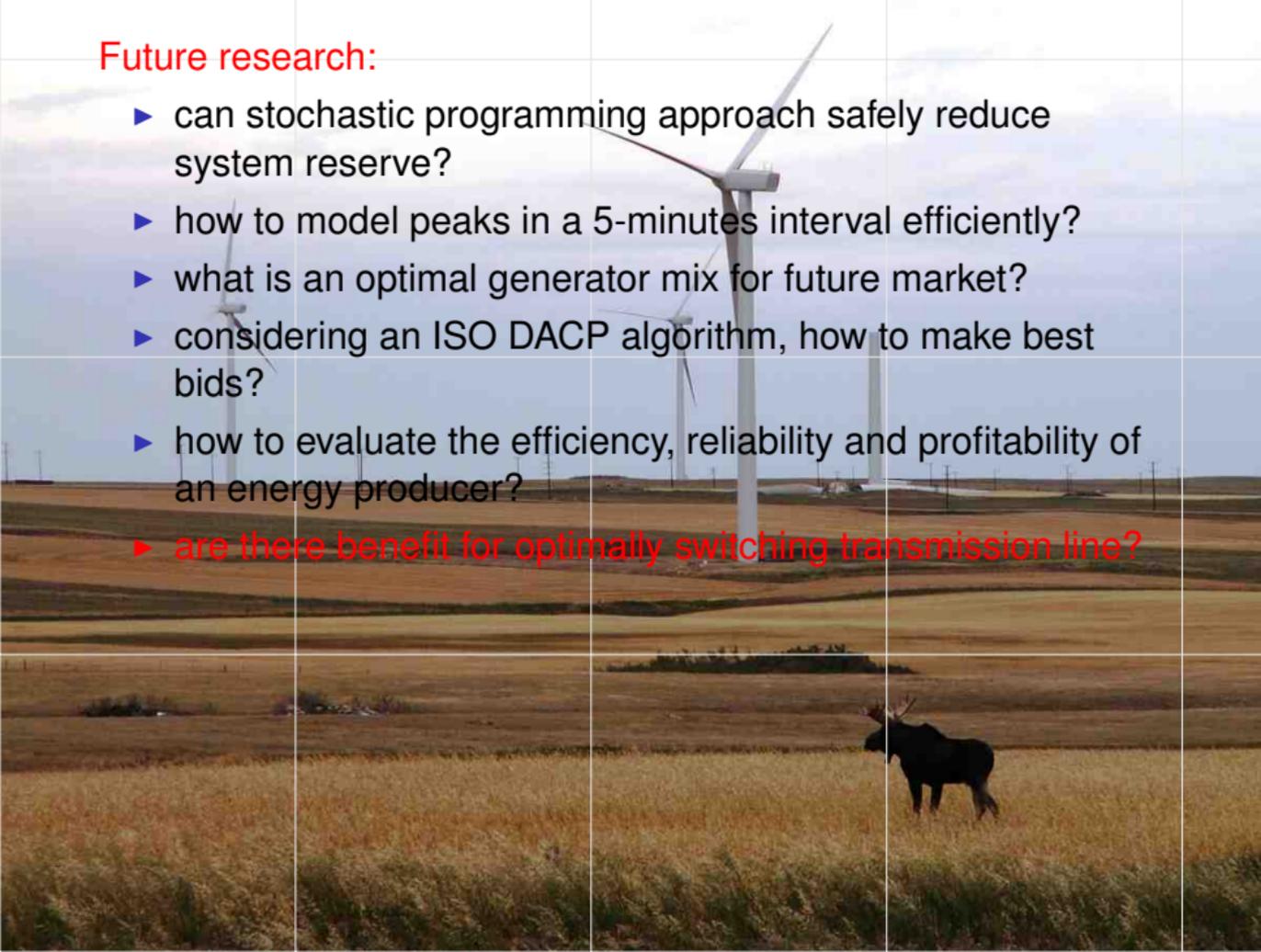
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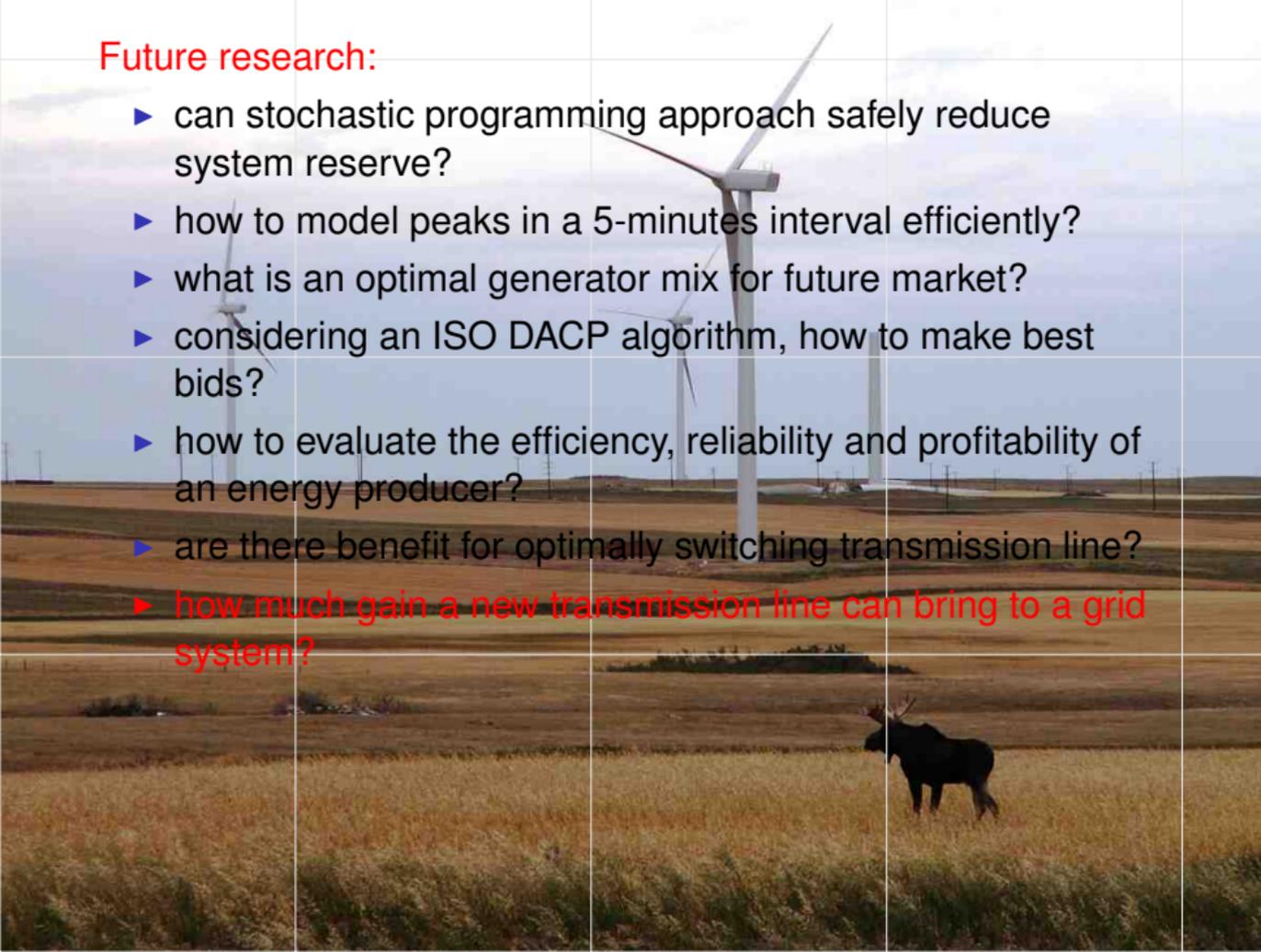
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- ▶ are there benefit for optimally switching transmission line?
- ▶ how much gain a new transmission line can bring to a grid system?



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