THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

CLAUDIO ALBANESE
University of Toronto

Integrability by Quadratures of Diffusion Equations

Co-author: Giuseppe Campolieti

In 1931, Kolmogorov posed the question of characterizing diffusion equations integrable by quadratures. The integrability problem was first addressed by Lie in 1887, who identifies the largest family solveable by group theoretical methods. Bluman in 1980 then showed that all models in Lie's family can be reduced to the Wiener process and have quadratic diffusions. In 1977, a 2-parameter family of integrable diffusions that doesn't fit into Lie's theory appeared in a finance paper by Cox and Ross. In this case, integrability is achieved by reduction to a Bessel process of non-integer dimension. In this talk, we describe a couple of 6-parameter families of integrable diffusion models which admit as particular cases all previously known solveable models in the literature. We outline two proofs, a non-constructive one based on the Cameron-Martin-Girsanov theorem, and a constructive one involving a kind of microlocal canonical transformations.

DEBORAH ALTERMAN
University of Colorado at Boulder

Diffractive Short Pulse Asymptotics for Nonlinear Wave Equations

The Slowly Varying Amplitude Approximation, which assumes a solution of the form of a slowly varying amplitude multiplying a rapidly varying phase, is a useful simplification when studying wave propagation equations such as Maxwell's Equations. Valid in the short wavelength limit, the SVAA leads to simpler equations for the amplitude function and removes the computational difficulty of resolving details on the short wavelength scale.

Technology has advanced to the point where ultrafast laser pulses can be produced which contain only a few wavelengths or even half a cycle. This creates the need for a new approximation to replace the SVAA when deriving simpler profile equations and performing computations. I will present a rigorously justified approximation, valid for short pulses, which leads to qualitatively different descriptions even in the linear case. Preliminary results indicate that this approach may describe features seen in short pulse experiments but not predicted by the Slowly Varying Amplitude Approximation. (Joint work with J. Rauch)

CLAUDE BARDOS
Université de Paris and Ecole Normale Spérieure - Cachan

Deterministic and random approaches to the Derivation of kinetic and macroscopic equations

Hierarchies of equations play an important role in the derivation of 'coarser' models from systems of equations which describe the details of microscopic dynamics. This is in particular the case for the derivation of the equations of kinetic theory from Liouville equations for the motion of the particles of a gas. In this contribution the emphasis is placed on the role of the randomness. One model is given which shows that the introduction of this randomness is compulsory to obtain the natural kinetic equation of motion.

OLEG I. BOGOYAVLENSKIJ
Queen's University

New symmetries of the magnetohydrodynamics equilibrium equations

Infinite dimensional Lie group G of symmetries of the magnetohydrodynamics equilibrium equations is introduced. The Lie group G depends upon the topology of the magnetic surfaces for a given equilibrium and is parametrized by two arbitrary smooth functions on the corresponding graph Γ.

RUSTUM CHOKSI
Simon Fraser University

Nonlocal variational problems and pattern morphologies in magnetic materials and diblock copolymers

Variational principles involving nonlocal terms are well- established for governing certain pattern morphologies in ferromagnetism and type-I superconductivity. Recently, a nonlocal functional has also been introduced to describe microphase separation in diblock copolymer melts. In this talk, I will address some basic issues surrounding the variational problems and associated pattern morphologies, and then discuss the application of some new mathematical tools. I will focus mostly on domain structures in uniaxial ferromagnets and microphases of diblock copolymers.

NICK ERCOLANI
University of Arizona

Landau Theory for Irrotational Vector Fields

The singular perturbation of the potential energy $\int(1-u^2)^2$ by $\epsilon^2|\nabla u|^2$ is a classical model for phase transitions. The extension of this problem from scalar fields u to gradient vector fields has until recently resisted analysis. In this talk we will review some recent analysis that has been done on singular limits of one such vector model in which the vector fields are constrained to be gradient. The physical and numerical motivations for this problem, coming primarily from the modelling of defects in pattern formation, will also be briefly described.

STEPHEN GUSTAFSON
Courant Institute

Dynamics in Ferromagnets

A class of nonlinear evolution equations (Landau-Lifshitz equations) for maps into the two-sphere arises in the study of ferromagnets. We discuss some basic (and largely open) problems associated with these PDE, including well- posedness, and the stability and dynamics of topological solutions (magnetic vortices or bubbles).

TOM HURD
McMaster University

Some nonlinear partial integro-differential problems arising in finance

I will give a brief review of the portfolio optimization problem in a general class of market models. We will see that the Hamilton-Jacobi-Bellman equation becomes a nonlinear partial integro-differential equation. After examining a number of exact solutions, I will describe a picture of the asymptotic behaviour of the general solution.

LEV KAPITANSKI
Kansas State University

Strichartz estimates for Schrödinger equations

In this talk I will discuss recent results on Strichartz estimates for Schrödinger equations with variable coefficients.

NICHOLAS KEVLAHAN
McMaster University

An adaptive wavelet method for calculating turbulent flow in complex geometries

Co-author: O. Vasilyev

Adaptive wavelet methods have recently been developed to solve the Navier-Stokes equations at high Reynolds numbers (e.g. Schneider, Farge & Kevlahan 1997; Vasilyev 1997). The adaptive wavelet method is appropriate to turbulence since the wavelets (which are localized in both space and scale) adapt the numerical resolution naturally to the intermittent structure of turbulence at small scales. The wavelet method thus allows turbulent flows to be calculated with a greatly reduced number of modes with little loss in accuracy. Furthermore, the computational cost of the algorithm is independent of the dimensionality of the problem and is $O(N)$, where N is the total number of collocation points actually used.

Parallel to the development of efficient wavelet codes for turbulence, we have been investigating the use of the Brinkmann equation to simulate the presence of arbitrarily complex solid boundaries (which may be moving in time) (Kevlahan & Ghidaglia 1999). This technique allows boundary conditions to be enforced to a specified precision, without changing the numerical method (or grid) used to solve the equations. The main advantage of this method, compared to other penalization type methods, is that the error can be estimated rigorously in terms of the penalization parameter. It can also be shown that the solution of the penalized equations converges to the exact solution in the limit as the penalization parameter tends to zero.

In this paper we describe the combination of the two approaches: a collocation adaptive wavelet method to solve the Navier– Stokes equations with a Brinkmann penalization to impose solid boundaries. This technique is applied first to Stokes flow (to test convergence and accuracy) and then to a variety of two-dimensional flows. The extension to three dimensions should be straightforward. The combination of these two methods has the potential to allow for the first time direct simulations of turbulent flows in geometries of engineering interest. Fluid-structure interactions are a natural application of this technique, and they are currently under investigation.

IZABELLA LABA
University of British Columbia

Recent work on the Kakeya conjecture

A Besicovitch set is a subset of R^n which contains a unit line segment in each direction; the Kakeya conjecture states that such sets must have dimension n. This seemingly innocent problem turns out to lie at the core of several basic open questions in multidimensional Fourier analysis. Attempts to resolve it have involved a mixture of analysis, combinatorics, geometry, and (more recently) additive number theory. In this talk I will describe the general background and some of the recent work on the problem.

JOCELINE LEGA
University of Arizona

Patterns and their dynamics

I will review pattern formation in extended systems and discuss how such structures can be modeled in terms of envelope equations near a bifurcation threshold. Other model equations, such as the Cross-Newell equation for almost-periodic patterns and the Swift-hohenberg equation, a generic pattern-forming model, will also be considered. I will present some applications of these models and discuss some open questions.

ANA-MARIA MATEI
McMaster University

First eigenvalue of the p-Laplacian

We describe the relationship between the first eigenvalue of the p-Laplacian on a compact riemannian manifold and other geometrical invariants of the manifold. We give a precise estimate for the first eigenvalue of the p-Laplacian in the case of a family of hyperbolic surfaces of fixed genus.

ROBERT MCCANN
University of Toronto

Optimal Transportation — from Monge and Kantorovich to Beckmann and Beyond: Uniqueness and Transport Density

This lecture concerns a classical optimization problem formulated by Monge in 1781. Motivated by economics, the problem is described as follows: Given a distribution f of iron mines throughout the countryside, and a distribution g of factories which require iron ore, decide which mines should supply ore to each factory in order to minimize the average distance transported. Taking the mines and factories may be distributed continuously throughout Euclidean space — or on a Riemannian manifold with boundary — yields a problem with rich connections to geometry and non-linear PDE.

This talk outlines joint works with Luis Caffarelli and Mikhail Feldman, and independent contributions by Ambrosio, Trudinger and Wang. These begin with a new approach to constructing solutions for the problem in the form of mappings from the manifold to itself. Although such solutions are far from unique, we give a geometrical monotonicity criterion singling out a unique map. Furthermore, a local definition is given for the transport cost density associated to each optimal map. All optimal maps are then shown to lead to the same transport density $a \in L^1(M)$, which itself solves a related economic optimization problem formulated by Beckmann (1952).

DMITRY E. PELINOVSKY
McMaster University

Normal forms for nonlinear resonance of embedded solitons

I review the concept and applications of embedded eigenvalues in various spectral problems associated with optical solitons. An embedded soliton (not an embedded eigenvalue!) occurs in a nonlinear PDE problem when a localized nonlinear mode coexists with the linear wave spectrum. A normal form for nonlinear resonance of embedded solitons is derived for a coupled two-wave system that generalizes the second-harmonic-generating model. It occurs as a result of a co-dimension one bifurcation of nonlocal wave solutions. Nonlinearity couples the embedded soliton and the linear wave spectrum and induces a one-sided radiation-driven decay of embedded solitons. The normal form shows that the embedded soliton is semi-stable, i.e., it survives under perturbations of one sign, but is destroyed by perturbations of the opposite sign. When a perturbed embedded soliton sheds continuous wave radiation, the radiation amplitude is generally not minimal. The analytical theory is confirmed by numerical computations of the Coupled two-wave system.

KEITH PROMISLOW
Simon Fraser University

Modulational Stability via Renormalization Methods for Patterns in forced dispersive systems

We study slow modulation of patterns, or quasi-steady solutions, in forced dispersive systems as arise in models of optical parametric processes. The linearization about a pattern yields a family of non-self adjoint operators. The localization of the point spectrum of these operators requires new techniques. Moreover we obtain a rigorous decomposition of the flow near the pattern from a renormalization group method, which permits us to overcome not only the weak smoothing properties of the linear semi- groups, but also their possible secular behavior. We obtain a family of ODEs which describe the evolution of the pattern and provide estimates on the error.

JUAN M. RESTREPO
University of Arizona

Soliton Resolution and Bound States in Nonlinear Dispersive Wave Equations

Integrability is a property of certain partial differential equations that is strongly associated with the resolution of data into a train of solitary waves. I will present numerical evidence that suggests that integrability is neither a necessary or sufficient condition for solitary wave resolution.

Separately, the time dependent numerical solutions to certain equations which have competing dispersive effects can lead to the appearance of bound state solutions. I will show our work, to date, on equations such as the quintic KdV and the Benjamin Equation, which show the existence of these bound states and that suggests that the competition of dispersive effects is crucial to their existence.

This is work is a collaboration with Jerry Bona and James Hyman.

ISRAEL MICHAEL SIGAL
University of Toronto

Non-radial solutions of the Ginzburg-Landau equation

Abstract unavailable

WALTER STRAUSS
Brown University

Electromagnetic Instability in a Collisionless Plasma

A collisionless plasma is modeled by the Vlasov-Maxwell equations, also called the collisionless Boltzmann equation with an internal field. There is no dissipation. There exist many kinds of steady states, including homogeneous states, BGK states and magnetic states. Some are stable and some unstable. How do we distinguish the stable ones? There has been a lot of progress on this question recently.

In this talk I will focus on a homogeneous state that is close to a maxwellian (=gaussian) but is anisotropic. Such a state is unstable under certain electromagnetic perturbations but is stable under purely electrostatic perturbations. The instability requires the spatial period to be larger than a certain precise critical value.

PIERRE-LOUIS SULEM
Centre National de la Recherche Scientifique (CNRS)

Numerical evidence of Alfvn-wave collapse in dispersive magnetohydrodynamics Co-authors: D. Laveder and T. Passot

The three-dimensional dynamics resulting from the transverse instability of a monochromatic Alfvn wave propagating along an ambient magnetic field is studied by direct numerical integration of the Hall-MHD equations.

The transverse collapse of a small-amplitude pump leading to the formation of intense magnetic filaments parallel to the ambient field is observed, which validates the predictions of the two-dimensional nonlinear Schrdinger equation and of more general amplitude equations that retain the coupling to the low-frequency magnetosonic waves. This picture is obtained in spite of the presence of other possibly linearly-dominant instabilities that can distort the above structures. In computational boxes including a large number of pump wavelengths, an early arrest of the collapse is possible under the effect of quasi-transverse instabilities that drive magnetosonic waves and also prescribe the directions of the filaments.

When the pump amplitude is increased, the local enhancement of the transverse magnetic field intensity is significantly reduced, but strong gradients develop. For pumps of moderate amplitude and sufficiently long wavelength, helicoidal magnetic filaments can nevertheless form, with a flattened cross section and a pitch equal to the pump wavelength, while the density and the longitudinal fields produce oblique shocks. Simultaneously, a turbulent flow develops, whose longitudinal spectrum is concentrated at large scales, well separated from the Alfvn wave. After averaging along the longitudinal direction on the scales smaller than or comparable to the pump wavelength, the resulting quasi two-dimensional flow is almost incompressible and its dynamics governed by the reduced-MHD equations.

AGNES TOURIN
University of Toronto

Stochastic control and viscosity solutions in Mathematical finance

Many interesting questions raised in mathematical Finance are adequately modelled as stochastic control problems. We will describe briefly the celebrated model of portfolio management posed and solved by Merton in 1969 and will also present its extension to a market with transaction costs. In addition we will explain the connection between these models and the theory of Black and Scholes.

We will then propose a new model in International Finance incorporating the concept of political risk. This is a joint work with J. Hodder and T. Zariphopoulou which relies on the Theory of viscosity solutions providing a notion of weak solution of the dynamic programming equation satisfied by the value function of the stochastic control problem.