THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

MIGUEL ABREU
The Fields Institute and Institute Superior Tecnico, Lisbon

Kähler metrics on toric orbifolds

In this talk I will describe a symplectic approach to Kähler geometry on toric orbifolds, based on the existence of action-angle coordinates. The main result of this approach is an effective parametrization of all invariant Kähler metrics on a symplectic toric orbifold, via smooth functions on the corresponding moment polytope.

As an application I will give a simple explicit description of recent work of R. Bryant, producing interesting new families of extremal Kähler metrics. In particular, one obtains an extremal Kähler metric on any weighted projective space.

SELMAN AKBULUT
Michigan State University

Lefschetz Fibrations on Compact Stein Surfaces

I will talk on my recent work with B. Ozbagci. I will give a topological proof of the fact that a compact Stein manifold is nothing more than a Lefschetz fibration over a disk with bounded fibers, and give a natural topological method of compactifying Stein manifolds to Symplectic manifolds, and discuss its consequences.

VESTISLAV APOSTOLOV
Université du Québec a Montréal

The curvature and the integrability of almost Kahler manifolds

In the space of all riemannian metrics compatible with a given symplectic form we look for metrics having certain curvature properties. The non-Kahler examples turn out to have a much more rigid geometric structure than expected, leading to complete classification results.

DENIS AUROUX
Ecole Polytechnique, Palaiseau

Projective maps and invariants of symplectic manifolds

The aim of this talk will be to explain how the topology of compact symplectic manifolds can be investigated using maps with values in projective spaces, and in particular maps to $\mathbb{CP}^2$. Focusing particularly on the four-dimensional case, we will introduce monodromy invariants and discuss some applications.

FRÉDÉRIC BOURGEOIS
Stanford University

A Morse-Bott approach to contact homology

Contact homology was introduced by Eliashberg, Givental and Hofer. In this theory, we count holomorphic curves in the symplectization of a contact manifold, which are asymptotic to isolated periodic Reeb orbits. The usefulness of contact homology has already been demonstrated by several computations for certain contact manifolds.

Unfortunately, these computations are limited and uneasy, because of the assumption that the periodic Reeb orbits must be nondegenerate (and, in particular, isolated). In other words, even when the contact manifold admits a natural and very symmetric contact form, this contact form has to be perturbed before starting the computation. As a consequence of this, the Reeb flow becomes rather chaotic and hard to study. Moreover, since the Cauchy- Riemann equation is perturbed as well, it is virtually impossible to compute the moduli spaces of holomorphic curves.

In this talk, we explain the construction of Morse-Bott contact homology for contact forms with nonisolated closed Reeb orbits, and satisfying certain Morse-Bott conditions. The Morse-Bott contact homology is isomorphic to the usual contact homology. We illustrate how this Morse-Bott construction is useful to actually compute contact invariants with several examples.

YURI CHEKANOV
Moscow Centre for Continuous Math Education

Proof of Arnold's four-cusp conjecture

(Joint work with P. Pushkar)

The four cusp conjecture formulated by V. I. Arnold about 10 years ago is as follows. Let V_t, $t \in [0,1]$, be a generic smooth family of co-oriented wavefronts in the plane such that V_0, V_1 are embedded circles with opposite co-orientations, and for each t the front V_t has no oriented self-tangencies. Then there exists $t_0 \in [0,1]$ such that V_{t_0} has at least four cusp points. The proof of this conjecture combines the classical Hurwitz theorem with studying combinatorics of wavefronts.

OCTAV CORNEA
Universite de Lille 1

Hopf invariants and periodic orbits of Hamiltonian flows

In this talk we discuss some ways to detect periodic orbits of autonomous Hamiltonian flows on non-compact regular hypersurfaces.

A first task that is already delicate in this setting is to produce bounded orbits of such flows. Fix a Hamiltonian $H : M \to \mathbf{R}$ with M symplectic but non-compact and let γ_H be the gradient flow of H with respect to some Riemannian metric on M and let X_H be the Hamiltonian flow of H.

The key idea is to translate certain algebraic topological properties of an index pair (in the sense of the Conley index) (N_1, N_0) of an isolated invariant set of γ_H into recurrence properties of X_H. It is useful to note that such index pairs (N_1, N_0) are reasonably easy to construct for gradient flows.

When H is generic our results show that if the relative homology $H_*(N_1, N_0; \mathbf{Z})$ has torsion or if a certain differential in the $\Omega_*^{fr}(\Omega M)$- Atiyah-Hirzebruch-Serre spectral sequence associated to the fibration induced over the pair (N_1, N_0) from the path loop fibration $\Omega M \to PM \to M$ does not vanish, then X_H has some non-trivial periodic orbits ($\Omega_*^{fr}(\Omega M)$ is the framed bordism ring of the pointed loop space of M, ΩM).

MICHAEL ENTOV
Weizmann Institute of Science

Symplectic topology and geometry of conjugacy classes in Lie groups

The talk will concern a new application of symplectic topology to a generalization of the following old problem: given eigenvalues of two unitary matrices A and B, describe possible eigenvalues of the product AB.

DMITRY FUCHS
University of California, Davis

Invariants of Legendrian mirror torus knots

Legendrian and transverse minor torus knots in the standard contact space has been recently clarified by Etnyre-Honda and Birman-Wrinkle. We can study their invariants (Bennequin numbers and Chekanov-Eliashberg invariants)to show that their possible values and behaviour contradict some natural conjectures and give rise to some others.

KENJI FUKAYA
Kyoto University

Homotopical algebra, Floer homology and Mirror symmetry

(This talk is based on joint works with Oh, Ohta, Ono)

Relation of rational homotopy type of a manifold M to its De Rham complex (differential graded algebra = DGA) $\Omega(M)$ is classical after the work by Quillen and Sullivan. In case when M is a Kahler manifold, the Kodaira-Spencer deformation theory of complex structure of M is controlled by the homotopical algebra of the DGA $\Omega(M;TM)$, the Dolbault complex of M with coefficient in the tangent bundle. Especially, the infinitesimal deformation theory of complex structure of M is controlled by the higher massey product which is nothing but the (rational) homotopy type of the DGA $\Omega(M;TM)$. The purpose of this talk is to explain its "quantization". Namely first an noncommutative generalization of the notion of DGA. (Note the DGA's appeared above are graded commutative.) We develop a rational homotopy theory of (non commutative) DGA. (Actually the homotopy category of non commutative DGA is equivalent to the (rational) homotopy category of A_∞ algebra. In case when L is a Lagrangian submanifold of a complex manifold, we can associate a (rational) homotopy type of an A_∞ algebra. The deformation theory of it is closely related to the Floer homology (its family version also) and to the deformation theory of the Lagrangian submanifold itself. This deformation theory together with Floer cohomology is expected to be the mirror of the deformation theory and cohomology theory of (objects of the derived category of) the coherent sheaves of the mirror.

The contents of the talk is based on our recent preprint, Lagrangian intersection Floer theory - Anomaly and obstruction, (the 2000 Dec version). However there are several points we made progress after that which will also be included in the talk.

MICHAEL HUTCHINGS
Stanford University

Periodic Floer homology

Let f be an area-preserving surface diffeomorphism, and let Y be the mapping torus of f. We define a new version of symplectic Floer homology, which we call "periodic Floer homology" (joint work with Michael Thaddeus). This is the homology of a chain complex in which the chains are generated by unions of periodic orbits of f, and the differential counts embedded pseudoholomorphic curves in R times Y. Our original motivation for studying this theory is that it is conjectured to agree with the Seiberg-Witten Floer homology of Y. Another motivation for studying this theory is that it has a formal analogue for a contact 3-manifold Y, which is a variant of the symplectic field theory of Eliashberg, Givental, and Hofer.

SERGEI MERKULOV
University of Glasgow

Extended Kaehler cone

We study geometric structures canonically induced on the dg moduli spaces which represent deformation functors in complex and symplectic geometry. In particular, we discuss solutions of WDVV equations arising from semi-infinite A-variations of Hodge structure over extended Kaehler cone, and compare these A-variations with Barannikov's B-variations for Monge-Ampere manifolds.

YONG-GEUN OH
University of Wisconsin-Madison

Floer theory, length minimizing property of geodesics and a new invariant norm on the Hamiltonian diffeomorphism group

We illustrate usage of the chain level Floer theory, which is the Floer theoretic version of classical mini-max theory for indefinite functionals, in the study of length minimizing property of geodesics on the Hamiltonian diffeomorphism group. As an example, we prove that any quasi-autonomous Hamiltonian path is length minimizing in its homotopy class with fixed ends, as long as it does not allow any periodic orbits of period less than one. This generalizes the results on the autonomous case by Hofer ($\mathbb{C}^n$) and by McDuff-Slimowitz (in general). If time permits, we will also explain how to use this chain level Floer theory to construct a new invariant norm of Viterbo's type on the Hamiltonian diffeomorphism group of arbitrary closed symplectic manifolds.

KAORU ONO
Hokkaido University

Lagrangian intersection Floer theory and deformation of Langrangian submanifolds.

I will give a report on a joint work with Kenji Fukaya, Yong-Geun Oh and Hiroshi Ohta. We defined obstruction classses for defining Floer homology for Lagrangian intersection and constructed Floer homology when all the obstruction classes vanish. In fact, this construction depends on several additional choices. We also constructed what we call a filtered A_∞-algebra associated to a Lagrangian submanifold, in which obstruction classes are formulated. Dependence on additional choices is also controlled in terms of this algebra. (In this work, Lagrangian submanifolds are assumed to be "relatively spin".)

PETYA PUSHKAR
University of Toronto

Morse theory for generating families and invariants of Legendrian knots

Suppose that a Legendrian knot Λ in the standard contact space $J^1(\mathbf{R})$ (or $J^1(S^1)$) can be defined by a generating family $F : \mathbf{R} \times W \to \mathbf{R}$ (or $F : S^1 \times W \to \mathbf{R}$). Applying to the generating family F one-parametric version of Morse theory, we construct certain combinatorial structures on the wavefront of Λ. These structures, in particular provide invariants of Legendrian knots.

YULI B. RUDYAK
Stanford University

On Lusternik–Schnirelmann category and the Arnold conjecture

Given a smooth manifold M, let $c(M)$ denote the minimal number of critical points of a smooth real-valued function on M. Let (M, ω) be a closed symplectic manifold, and let $\varphi : M \to M$ be a Hamiltonian symplectomorphism of M. The well-known Arnold conjecture claims that the number $\operatorname{Fix}\varphi$ of fixed points of φ is at least $c(M)$, i.e. $\operatorname{Fix}\varphi \geq c(M)$. Floer and Hofer proved that, for manifolds M with $\pi_2(M) = 0$, the number $\operatorname{Fix}\varphi$ is at least the cup-length of M. So, since $c(M)$ is at least the cup-length of M, we have here a weak form of the Arnold conjecture. Based on Floer–Hofer analytical results, and using some additional topological arguments, we prove that, for manifolds M^{2n} with $\pi_2(M) = 0$, $c(M) = 2n + 1$ and $\operatorname{Fix}\varphi \geq 2n + 1$. In particular, the original Arnold conjecture holds for such manifolds.

MISHA VERBITSKY
Moscow Center of Continuous Math Education

Deformation of non-compact holomorphic symplectic manifolds

(Joint work with D. Kaledin)

We study deformations of a non-compact holomorphically symplectic manifold M. A version of Bogomolov-Tian-Todorov is proven. Whenever $H^1(O_M) = H^2(O_M) = 0$, the formal deformation space of M is a formal completion of $H^2(M)$.