

# WEAK HOPF ALGEBRAS AND SINGULAR SOLUTIONS OF QUANTUM YANG-BAXTER EQUATION

FANG LI

ABSTRACT. We investigate a generalization of Hopf algebra  $\mathfrak{sl}_q(2)$  by weakening the invertibility of the generator  $K$ , i.e. exchanging its invertibility  $KK^{-1} = 1$  to the regularity  $K\overline{K}K = K$ . This leads to a weak Hopf algebra  $w\mathfrak{sl}_q(2)$  and a  $J$ -weak Hopf algebra  $v\mathfrak{sl}_q(2)$  which are studied in detail. It is shown that the monoids of group-like elements of  $w\mathfrak{sl}_q(2)$  and  $v\mathfrak{sl}_q(2)$  are regular monoids, which supports the general conjecture on the connection between weak Hopf algebras and regular monoids. Moreover, from  $w\mathfrak{sl}_q(2)$  a quasi-braided weak Hopf algebra  $\overline{U}_q^w$  is constructed and it is shown that the corresponding quasi- $R$ -matrix is regular  $R^w \hat{R}^w R^w = R^w$ .

## 1. INTRODUCTION

The concept of a weak Hopf algebra as a generalization of a Hopf algebra was introduced and its characterizations and applications were studied in other works of the author's. A  $k$ -bialgebra<sup>1</sup>  $H = (H, \mu, \eta, \Delta, \varepsilon)$  is called a *weak Hopf algebra* if there exists  $T \in \text{Hom}_k(H, H)$  such that  $id * T * id = id$  and  $T * id * T = T$  where  $T$  is called a *weak antipode* of  $H$ . This concept also generalizes the notion of the left and right Hopf algebras.

The first aim of this concept is to give a new sub-class of bialgebras which includes all of Hopf algebras such that it is possible to characterize this sub-class through their monoids of all group-like elements. It was known that for every regular monoid  $S$ , its semigroup algebra  $kS$  over  $k$  is a weak Hopf algebra as the generalization of a group algebra.

The second aim is to construct some singular solutions of the quantum Yang-Baxter equation (QYBE) and research QYBE in a larger scope. On this hand, a quantum quasi-double  $D(H)$  for a finite dimensional cocommutative perfect weak Hopf algebra with invertible weak antipode was built and it was verified that its quasi- $R$ -matrix is a regular solution of the QYBE. In particular, the quantum quasi-double of a finite Clifford monoid as a generalization of the quantum double of a finite group was derived.

In this paper, we will construct two weak Hopf algebras in the other direction as a generalization of the quantum algebra  $\mathfrak{sl}_q(2)$ . We show that  $w\mathfrak{sl}_2(q)$  possesses a quasi- $R$ -matrix which becomes a singular (in fact, regular) solution of the QYBE, with a parameter  $q$ . In this reason, we want to treat the meaning of  $w\mathfrak{sl}_q(2)$  and

---

1991 *Mathematics Subject Classification*. Primary 16W30, 81R50; Secondary 17B37, 57M25. Project (No. 19971074) supported by the National Natural Science Foundation of China.

<sup>1</sup>In this paper,  $k$  always denotes a field.

its quasi- $R$ -matrix just as  $\mathfrak{sl}_q(2)$ . It is interesting to note that  $w\mathfrak{sl}_q(2)$  is a natural and non-trivial example of weak Hopf algebras.

DEPARTMENT OF MATHEMATICS, ZHEJIANG UNIVERSITY (XIXI CAMPUS), HANGZHOU, ZHEJIANG  
310028, CHINA

*E-mail address:* `fangli@mail.hz.zj.cn`