THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

SIMONETTA ABENDA
CIRAM, University of Bologna, Italy

Modulation theory for the Camassa-Holm equation

Using the Whitham method, we derive the modulation equations for the Camassa-Holm equation. The modulation equations are hyperbolic and can be integrated through the so called hodograph transform. The equations are Hamiltonian with respect to a non-local Poisson bracket. A reciprocal transformation of the space-time maps the non-local Poisson bracket to the Poisson bracket of the Whitham equation for KdV.

DARIO BAMBUSI
Universit degli Studi di Milano

Birkhoff normal form for PDEs with Tame Modulus

I will present an abstract Birkhoff normal form theorem for Hamiltonian Partial Differential Equations. The theorem applies to semilinear equations with nonlinearity satisfying a property that will be called of Tame Modulus. The normal form can be used to deduce informations on the dynamics of the system. In particular, in the nonresonant case, one gets that any small amplitude solution remains very close to a torus for very long times. Moreover, one gets a long time estimate of higher Sobolev norms of the solution. The theorem applies to several concrete examples ranging from the nonlinear wave equation with Dirichlet or periodic boundary conditions in one space dimensions to some particular NLS and plate equations in d–space dimensions. Several tools to deal with applications will be presented in the talk by Benoit Grebert.

MASSIMILIANO BERTI
SISSA, Italy

Bifurcation of free vibrations for completely resonant wave equations

We prove existence of small amplitude, $2\pi/\omega$-periodic in time solutions of completely resonant nonlinear wave equations for any frequency ω belonging to a Cantor-like set of positive measure and for a generic set of nonlinearities. The proof relies on a suitable Lyapunov-Schmidt decomposition and a variant of the Nash-Moser Implicit Function Theorem.

S. ROY CHOUDHURY
University of Central Florida

Integrability characteristics and coherent structures of integrable two-dimensional generalizations of NLS type equations

A recent algorithmic procedure based on truncated Painlevé expansions is used to derive Lax Pairs, Darboux Transformations, Hirota Tau Functions, and various soliton solutions for integrable (2+1) generalizations of NLS type equations[1]. In particular, diverse classes of solutions are found analogous to the dromion, instanton, lump, and ring soliton solutions derived recently for (2+1) KdV Type Equations, the Nizhnik-Novikov-Veselov Equation, and the Broer-Kaup system.

If time permits, we shall also consider an algorithmic method for deriving the integrability characteristics of multicomponent integrable nonlinear PDEs via truncated Painlevé expansions. A similar method has been well-known for scalar integrable NLPDEs for several years, but no systematic procedure has existed for multicomponent systems. We shall demonstrate that if one uses only one singular manifold, and if one follows the concept of enforcing integrability at each step, then when the system is indeed integrable, one is led immediately and directly to the appropriate Lax Pair, although perhaps in nonlinear form. However, the problem now becomes one of finding a linearization of the nonlinear Lax pair thus found, which so far has had a solution in all cases. The method is illustrated for the NLS, Manakov, and Simple Harmonic Generation systems.

PERCY DEIFT
Courant Institute of Mathematical Sciences

Long-time asymptotics for solutions of the NLS equation with initial data in a weighted Sobolev space

The speaker will show how to use the steepest descent method to obtain precise asymptotics for solutions of the Cauchy problem for the defocusing NLS equation on the line. Special emphasis is given to the case where the initial data q_0 has minimal smoothness and decay. If the initial data is smooth and has rapid decay, absolute estimates suffice, but now one must use cancellations from oscillations.

Precise asymptotics for solutions of the Cauchy problem in the case that q_0 has minimal smoothness and decay, are needed,, in particular, in the solution of the perturbation problem for NLS. This is joint work with Xin Zhou.

HAKAN ELIASSON
Universit Paris 7

KAM for the non-linear Schroedinger equation

We shall discuss the so-called second Melnikov condition for the non-linear Schroedinger equation with periodic boundary conditions. (This is a report on a work in progress together with S. Kuksin.)

BENOIT GREBERT
Universite de Nantes

Birkhoff normal form for NLS and NLW

I will apply an abstract Birkhoff normal form theorem in infinite dimension recently obtained by D. Bambusi and I (and presented in the talk by D. Bambusi) to concrete examples of Hamiltonian PDEs. These concrete examples include the NLW equation and the NLS equation in one space dimension with Dirichlet or Periodic boundary conditions. In particular, in the Dirichlet case, one gets that any small amplitude solution remains close to a torus for very long times.
In the periodic case, resonances may appear and the result is more complicated; nevertheless, one gets a long time estimate of higher sobolev norms of the solution.
Our results also applies to a particular NLS equation in higher dimension (with a convolution potential).
I will focus on the verification of the hypothesis of the abstract theorem, in particular the nonresonances condition.

JOHN HARNAD
CRM and Concordia University

Hamiltonian theory of the general rational isomonodromic deformation problem

A number of special cases of systems of ordinary and partial differential equations generating deformations of a rational covariant derivative operator that preserve its generalized monodromy data are known to have a Hamiltonian structure. These include the Schlesinger systems, the Painlev equations, and the case of an operator of arbitrary rank with an arbitrary number of regular singular points, plus a single one with Poincare index 1. The underlying Hamiltonian structure in all these cases has been shown to coincide with the rational R-matrix structure used in the theory of integrable systems. The Hamiltonians are known to coincide with the logarithmic derivatives of the associated isomonodromic tau functions, and also to be spectral invariants of the associated rational connection matrix. These results will be extended to include the general, non-resonant rational isomonodromic deformation system on a punctured Riemann sphere, as well as systems over elliptic curves. If time permits, it will be shown how specific classes of solutions satisfying "Virasoro constraints" may be produced from the theory of generalized semi-classical orthogonal polynomials. The tau function for these solutions is explicitly related to the partition function of an associated random matrix model.

EDUARD-WILHELM KIRR
University of Chicago

Parametric resonance in NLS

Consider the 3-D cubic NLS with an attractive potential. Our result concerns the stability of the ground state under time periodic perturbations (nonlinear management). We show that, due to a resonant coupling with the radiation, the amplitude of the ground state decays polynomially in time and any solitary waves in the system are eventually destroyed. The result can be viewed as a "second order correction" to the time averaged dynamics which predicts the survival of a solitary wave.

SERGEI KUKSIN
Heriot-Watt University

KAM for PDEs in 1D and nD

I shall discuss recent progress in KAM for PDE, including new results of J. Bourgain, and results of H. Eliasson and myself.

NADER MASMOUDI
Courant Institute of Mathematical Sciences

The zero surface tension limit for the water wave problem

We consider two-dimensional water waves with surface tension. We prove that when surface tension goes to zero, we recover the water wave system without surface tension. This gives in particular a new proof of existence for the water wave system without surface tension.

MAUNG MIN-OO
McMaster University

Some asymptotic properties of Gaudin spin chains

After introducing the Gaudin spin chain as a quantum integrable system and the Bethe Ansatz as applied to this system, I will describe some recent work by A.Bourget, D.Jakobson, J.Toth and myself about the average distribution of zeros of Heine-Stieltjes polynomials in the thermodynamic limit.

N. SRI NAMACHCHIVAYA
University of Illinois at Urbana-Champaign

Nonstandard reduction of noisy dynamical systems

The focus of this work is the development of general techniques of stochastic averaging of randomly-perturbed four-dimensional integrable Hamiltonian systems. The integrable system here has certain nontrivial (yet generic) types of fixed points. Stochastic averaging makes use of these integrable structures to identify a reduced diffusive model on a space which encodes the structure of the fixed points and can have dimensional singularities. At these singularities, glueing conditions will be derived, these glueing conditions completing the specification of the dynamics of the reduced model. Qualitative dependence of statistical measures of the reduced system upon various coefficients can be studied by extensions of known techniques such as stochastic bifurcation theory.

JÜRGEN PÖSCHEL
University of Stuttgart

On the well-posedness of the KdV equation in weighted Sobolev spaces

We describe results about the well-posedness of the periodic KdV equation in weighted Sobolev spaces, which include for example Gevrey-type spaces. The argument is based on the existence of global Birkhoff coordinates for KdV.

DAVID SATTINGER
Yale University

Calogero-Françoise flows and Periodic Peakons

Motivated by the multipeakon Hamiltonian system discovered by Camassa and Holm, F. Calogero introduced a class of Hamiltonian systems of the form

$$H(x,m) = \sum_{j,k=1}^{d} m_j m_k F(x_j - x_k),$$

and showed that the resulting Hamiltonian system is completely integrable for $F(x) = \lambda + \mu \cos \nu x$. Subsequently he and Françoise proved complete integrability for $\lambda + \mu \cos \nu x + \mu' \sin \nu|x|$, and in the limiting case $\alpha + \beta|x| + \gamma x^2$.

We show that there is a close connection between the Calogero-Françoise flows and multipeakon flows of the Camassa-Holm equation with discrete periodic potential m. In both cases, the situation is mathematically similar to the case of multi-gap solutions of the periodic KdV equation. There is a Riemann surface which is invariant under the flow, and the flow is linearized by the Abel map on a hyperelliptic curve. For d particles, the associated invariant curve is hyperelliptic of genus $d - 1$.

The case $d = 2$, for which the associated curve is elliptic, is treated in detail. The flows are obtained explicitly in terms of Weierstrass elliptic functions (more precisely, in terms of the associated σ functions).

ISRAEL MICHAEL SIGAL
University of Toronto

Some mathematical questions related to the Bose-Einstein condensation

In this talk I review some recent rigorous results and formulate open mathematical problems related to the Bose-Einstein condensation. The latter was predicted in 1924 and discovered experimentally in trapped gases in 1995. It offers challenging and beautiful mathematical problems in Quantum Many-Body Theory and nonlinear PDEs.

JACEK SZMIGIELSKI
University of Saskatchewan

Multi-peakon solutions of the Degasperis–Procesi equation and almost Padé approximants

An inverse scattering approach is presented for computing n-peakon solutions of the Degasperis–Procesi equation (a modification of the Camassa–Holm shallow water equation). The associated non-self-adjoint spectral problem is a generalization of the Krein string to a third order case for which the n-peakon dynamics generates an isospectral deformation. As a result one obtains a nontrivial insight into the spectral properties of this non-self-adjoint problem. Moreover, the inverse problem can solved by a method generalizing the continued fraction solution of the inverse problem of the Krein string to a new situation involving simultaneous rational approximations, similar to Padé approximants, of two Weyl functions. This is joint work with H. Lundmark (Linköping, Sweden).

ALEXANDER TOVBIS
University of Central Florida and Fields Institute

Semiclassical (zero dispersion limit) solutions to the focusing NLS for a special class of the initial data with both soliton and solitonless cases

We calculate the leading order term of the solution of the focusing Nonlinear (cubic) Schroedinger Equation (NLS) in the semi-classical limit for a certain one-parameter family of initial conditions. This family contains both solitons and pure radiation. In the pure radiation case, our result is valid for all times $t \geq 0$. We utilize the Riemann-Hilbert Problem formulation of the inverse scattering problem to obtain the leading order term of the solution. Error estimates are provided. We also find the long time behavior of semi-classical solutions for solitonless cases within some range of time-small parameter dependence.

STEPHANOS VENAKIDES
Duke University

Rigorous results on semiclassical long-time focusing NLS

We will outline some of our results on semiclassical focusing NLS that include the existence and properties of a first breaking (nonlinear caustic) curve on the space-time plane and the leading behavior of the solution. We will then introduce new results on the long time behavior of the semiclassical solution. This is joint work with Alex Tovbis and Xin Zhou.

VITALI VOUGALTER
McMaster University

Spectra of positive and negative energies in the linearized NLS problem

We study the spectrum of the linearized NLS equation in three dimensions, in association with the energy spectrum. We prove that unstable eigenvalues of the linearized NLS problem are related to negative eigenvalues of the energy spectrum, while neutrally stable eigenvalues may have both positive and negative energies. We show how the negative index of the problem can be reduced by going to the proper constrained subspace.

EUGENE WAYNE
Boston University

Exponential averaging and the "pinning" of traveling waves in heterogenous media

Traveling waves can become "pinned" in heterogenous media. That is, the presence of small scale structures in the medium can cause a failure of propagation so that rather than having traveling waves one has stationary, spatially localized solutions. Such solutions may have useful applications in biological and physical systems. In joint work with K. Matthies (FU Berlin) we have used averaging methods to show that if pinning occurs in periodic media it can do so only for very small ranges of parameters meaning that in practice it may be quite hard to observe.

J. DOUGLAS WRIGHT
Fields Institute

Higher order corrections to KdV approximations

The KdV equation is frequently used to approximate solutions to non-linear dispersive systems, most notably that of surface water waves. While this approximation has been shown to be quite accurate, there are notable discrepencies from the predictions of the KdV model. We present a hierarchy of equations which correct for these discrepencies and discuss the validity of such models.

YINGFEI YI
Georgia Institute of Technology

Quasi-periodic standing waves in nonlinear Schrödinger equations

This talk is about the existence of quasi-periodic standing waves in 1D, cubic nonlinear Schrödinger equation with a periodic potential. It will be shown that there is a two-frequency quasi-periodic standing wave corresponding to each internal frequency of constant type which is larger than an explicitly given bound. This work, joint with S.-N. Chow, M. van Noort and M. A. Porter, is motivated by the study of BECs when considering only two body, mean field interactions in a dilute Bose-Einstein gas in 1D regime.

JIANGONG YOU
Nanjing University, China

Construction of quasi-periodic solutions via KAM theory

In this talk, we will present a generalized KAM theorem which can be applied to one dimensional Schrödinger equations with periodic boundary conditions and higher dimensional beam equations.