50 YEARS NONCLASSICAL PROTTER PROBLEMS FOR THE WAVE EQUATION

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In 1952 M. Protter formulated and studied boundary value problems for the wave equation, which are multidimensional analogues of Darboux-problems on the plane. More precisely, he studied these problems in a $3D$ domain Ω, bounded by two characteristic cones S_1 and S_2 and a plane region S_0. Many authors studied these problems using different methods, like: Wiener-Hopf method, special functions, a priori estimates, nonlocal regularization and others. It is shown in 1995 that for n in $\mathbb{N}$ there exists a right hand side function from $C^n(\bar{\Omega})$, for which the corresponding unique generalized solution belongs to $C^n(\bar{\Omega}\backslash O)$ and has a strong power-type singularity at the vertex O of the characteristic cone S_2. This singularity is isolated at the point O and does not propagate along the cone S_2. We will describe the exact behavior of the singular solutions at the point O and give some necessary and sufficient conditions for existence of solutions with fixed order of singularity. It states some exact a priori estimates for the singular solutions. Some nonlocal regularization of these strongly ill-posed problems will be given, too. Some $3D$ problems for changing type equations, connected with the above problems, will also be discussed.