Teleportation in ground state quantum computation

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Outline

1. Ground-state quantum computation

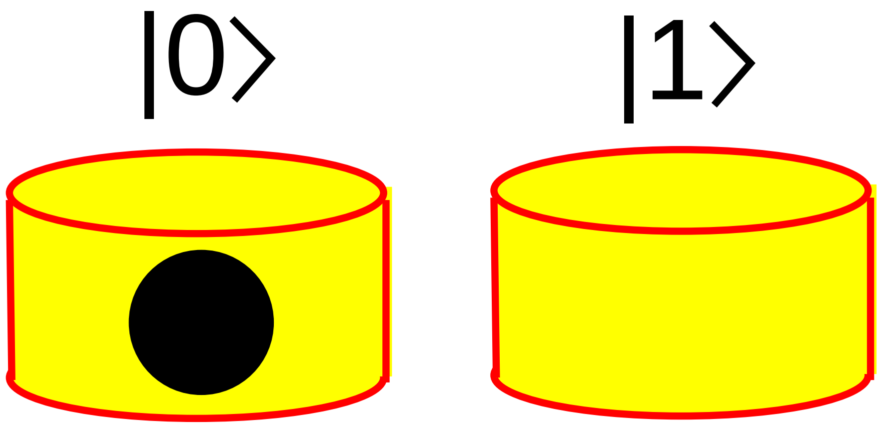
2. Decoherence and the energy gap

3. Teleportation and parallelization

4. Future prospects

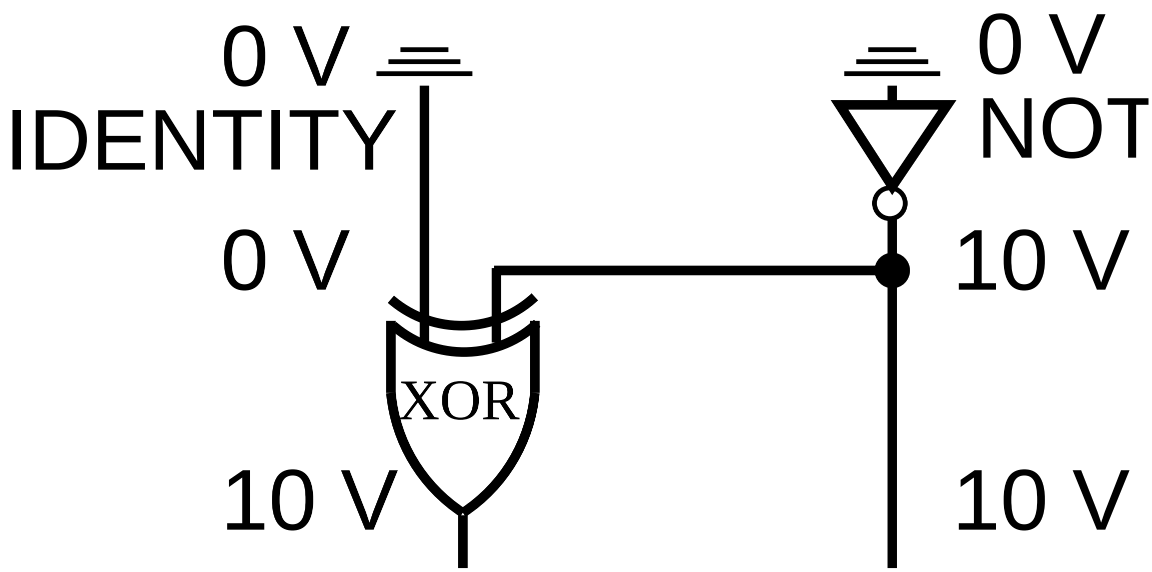
Quantum dot qubit

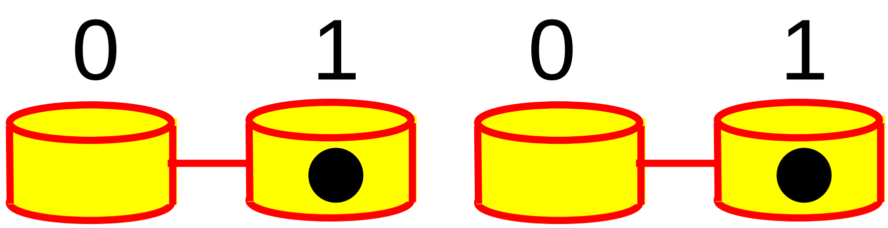
- Consider a single electron shared by two quantum dots



- This hypothetical realization of a qubit is convenient for illustrating the idea of ground state quantum computation

Classical computer vs quantum computer





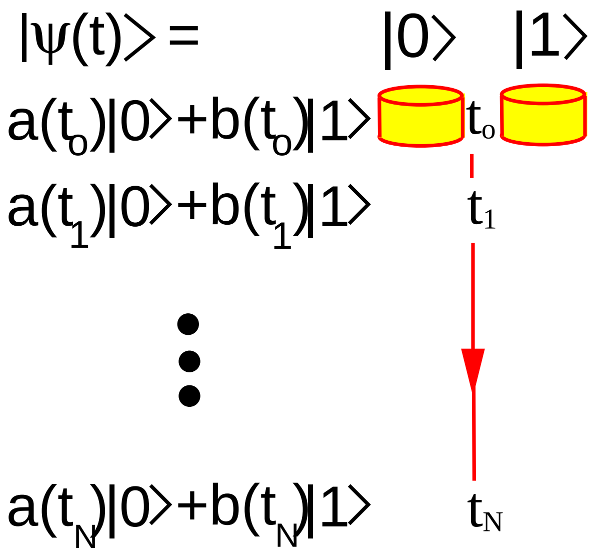
- Classical: computation is spread out in space each clock cycle; bits go to gates

- Quantum: computation is not spread out in space; gates go to bits

Quantum computation

- A quantum computation progresses in time through $N + 1$ time steps

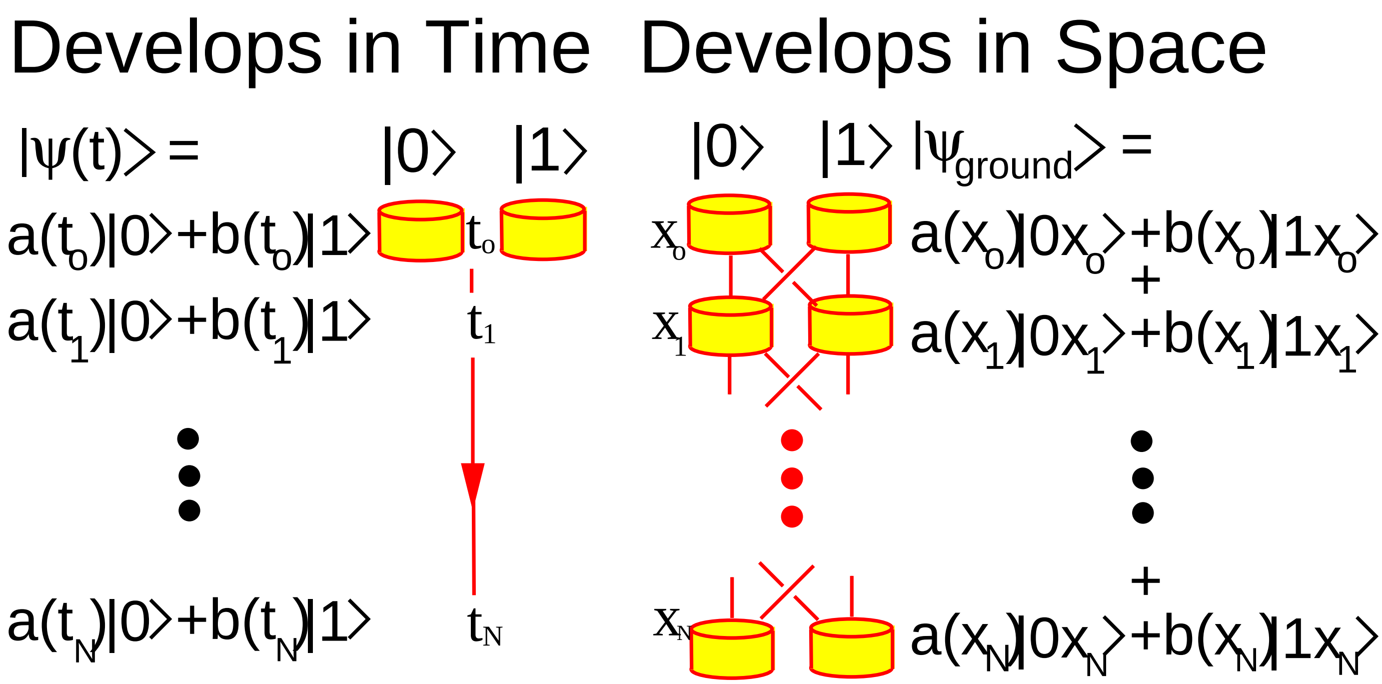
Develops in Time



- 2(N+1) complex numbers characterize the trajectory

Ground state quantum computation

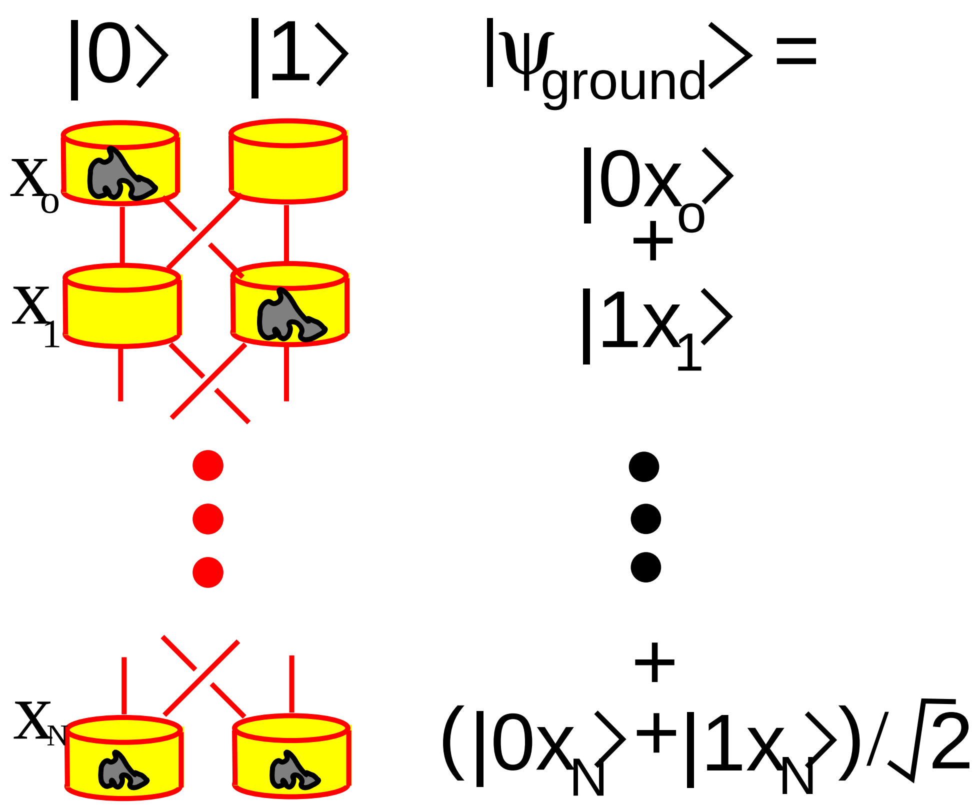
- Essential idea: replace a 2 dimensional qubit evolving over $(N+1)$ time steps with a time independent $2(N+1)$ dimensional qubit



- 2(N+1) complex numbers in each case
- The ground state of a single electron in the array contains the trajectory of the algorithm from input to output

Example ground state

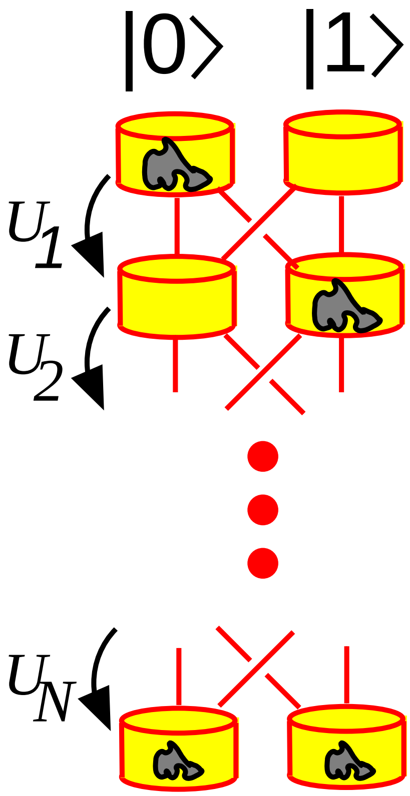
- Suppose algorithm takes input $|0\rangle$, applies a NOT gate, then several other gates, and eventually outputs $(|0\rangle + |1\rangle)/\sqrt{2}$.



Suitable (unnormalized) ground state

Example ground state

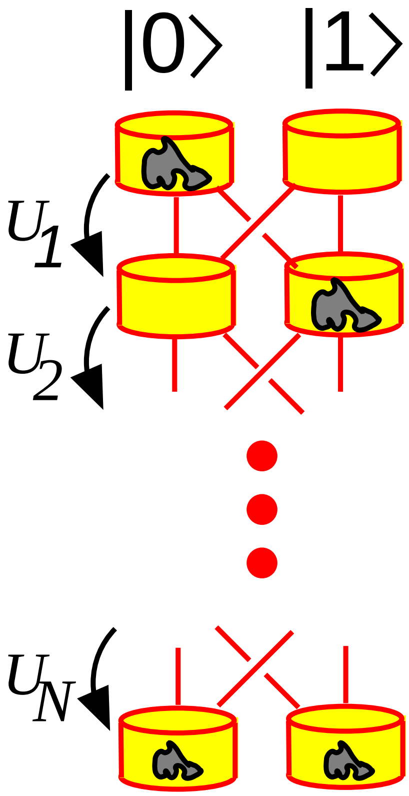
- Writing out the state as a column vector



$$\begin{bmatrix} 1 \\ 0 \\ \\ 0 \\ 1 \\ \vdots \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

Example ground state

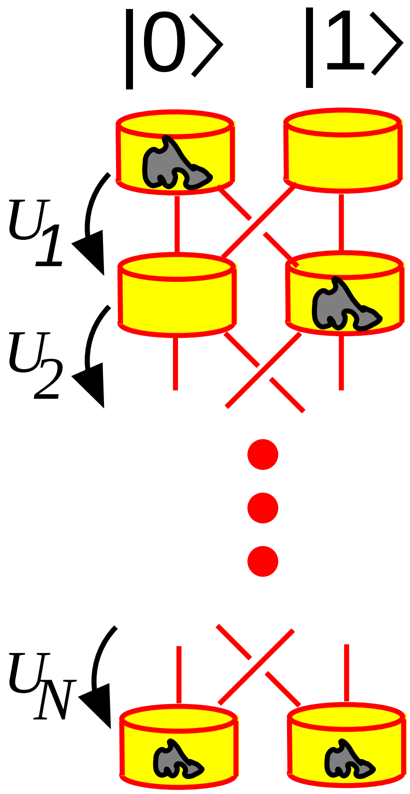
- Writing out the state as a column vector



$$
\begin{bmatrix} 1 \\ 0 \\ \\ 0 \\ 1 \\ \\ \vdots \\ \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}
=
\begin{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \\ U_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \vdots \\ U_N \ldots U_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix}
$$

- Can we design a Hamiltonian of on-site and tunneling matrix elements with the desired ground state?

Hamiltonian



This positive semi-definite Hamiltonian has the desired state as its ground state.

The gate U_i fixes the tunneling matrix elements between the two dots on row $i-1$ and the two dots on row i.

$$\epsilon \begin{bmatrix} I & -U_1{}^\dagger & & & \\ -U_1 & 2I & -U_2{}^\dagger & & \\ & & \ddots & & \\ & & & \ddots & \\ & & -U_{N-1}{}^\dagger & 2I & -U_N{}^\dagger \\ & & & -U_N & I \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ U_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \vdots \\ U_N \ldots U_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} = 0$$

Running a ground state quantum computation

1. Start with array of $2(N+1)$ quantum dots

2. Tune tunneling matrix elements between dots in accordance with algorithm

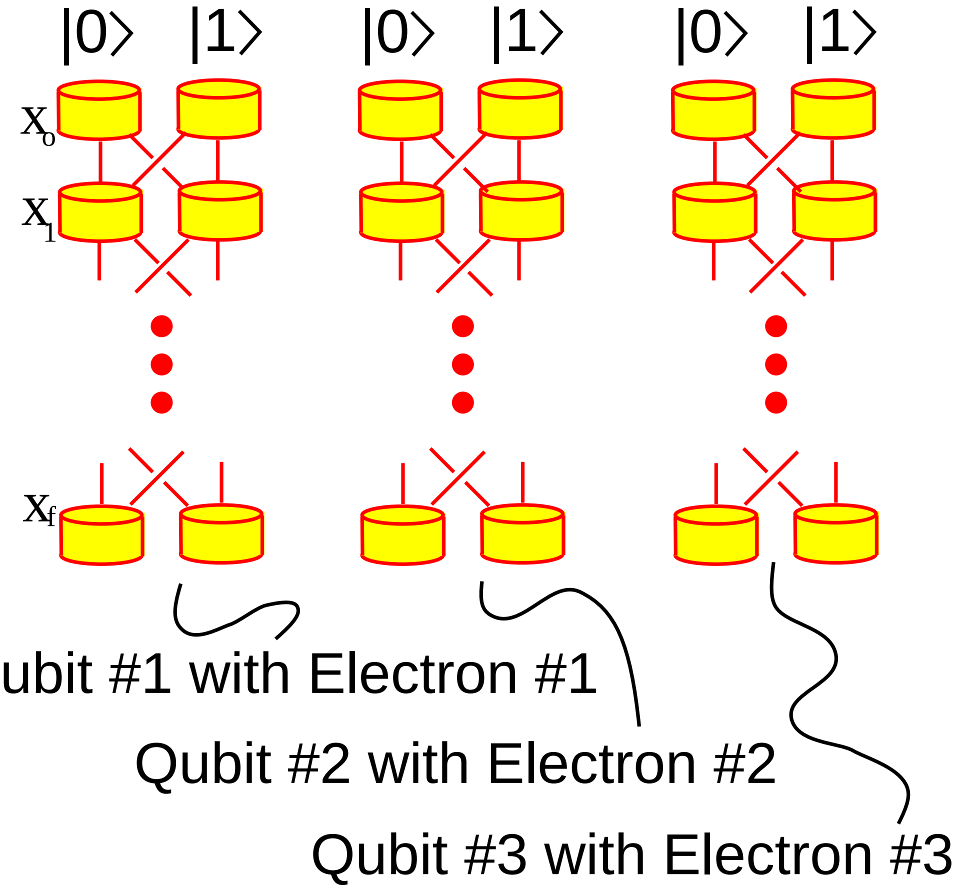
3. Place one electron in the array

4. Cool electron to the ground state

5. Output is the amplitude of the electron on bottom two dots of the computer

Multiple qubits

- So far, only a single qubit has been described, but multiple qubits are required for useful computation



- Including two-body interactions, arbitrary quantum computation algorithms can be run

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What about decoherence?

- An energy barrier to the first excited state defends against decoherence

- Time-dependent perturbations have to excite the computer out of its ground state to derail the computation

- Time-independent perturbations to the desired Hamiltonian distort the ground state

- Precise static Hamiltonian with large level spacings is required

Scaling of energy gap

For a general quantum computation, can prove that energy gap to first excited state:

- Has lower bound of $O(1/N^4)$
- Has upper bound of $O(1/N^2)$
- Independent of number of qubits

Strengths & weaknesses of ground-state approach

Strengths

- Energy barrier defends against decoherence
- The system does not require time-dependent control
- New algorithmic capabilities: Non-unitary development

Weaknesses

- A precise static Hamiltonian on a large Hilbert space is required $\rightarrow$ Vulnerable to implementation errors like time-dependent quantum computation
- Energy gap decreases with system size
- System must be cooled to ground state
- Can only measure classical result (like NMR QC)

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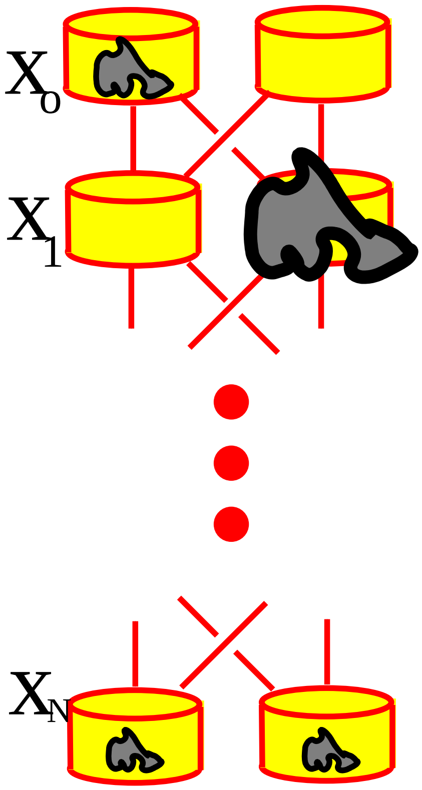
Non-unitary development

- Although we can mimic evolution in time by development down the array, we do not have to adhere to the same constraints

$$|0\rangle \quad |1\rangle \quad |\psi_{ground}\rangle = |0x_o\rangle + \lambda\,|1x_1\rangle + \cdots + (|0x_N\rangle + |1x_N\rangle)/\sqrt{2}$$



- Normalization from row to row can be controlled with small modifications to Hamiltonian

Quantum teleportation

- Can we exploit the non-unitary flexibility?

- Recall quantum teleportation

$$(U|0\rangle)\left(\frac{|0\rangle|0\rangle+|1\rangle|1\rangle}{\sqrt{2}}\right)$$
$$=\frac{1}{2}\left(\frac{|0\rangle|0\rangle+|1\rangle|1\rangle}{\sqrt{2}}\right)(U|0\rangle)$$
$$+\text{ other terms}$$

- Measure first two qubits in Bell basis to teleport unknown state into third qubit; works $(1/2)^2 = 1/4$ of the time

Quantum gates via teleportation

- Quantum gate applied via teleportation

$$(U_1 |0\rangle) \left(\frac{|0\rangle U_2 |0\rangle + |1\rangle U_2 |1\rangle}{\sqrt{2}} \right)$$
$$= \frac{1}{2} \left(\frac{|0\rangle |0\rangle + |1\rangle |1\rangle}{\sqrt{2}} \right) (U_2 U_1 |0\rangle)$$
$$+ \text{ other terms}$$

- Successful teleportation 1/4 of the time
- Using non-unitary gates, we can boost the chance of success
- This process can be used to parallelize quantum computation

Parallelizing quantum computation

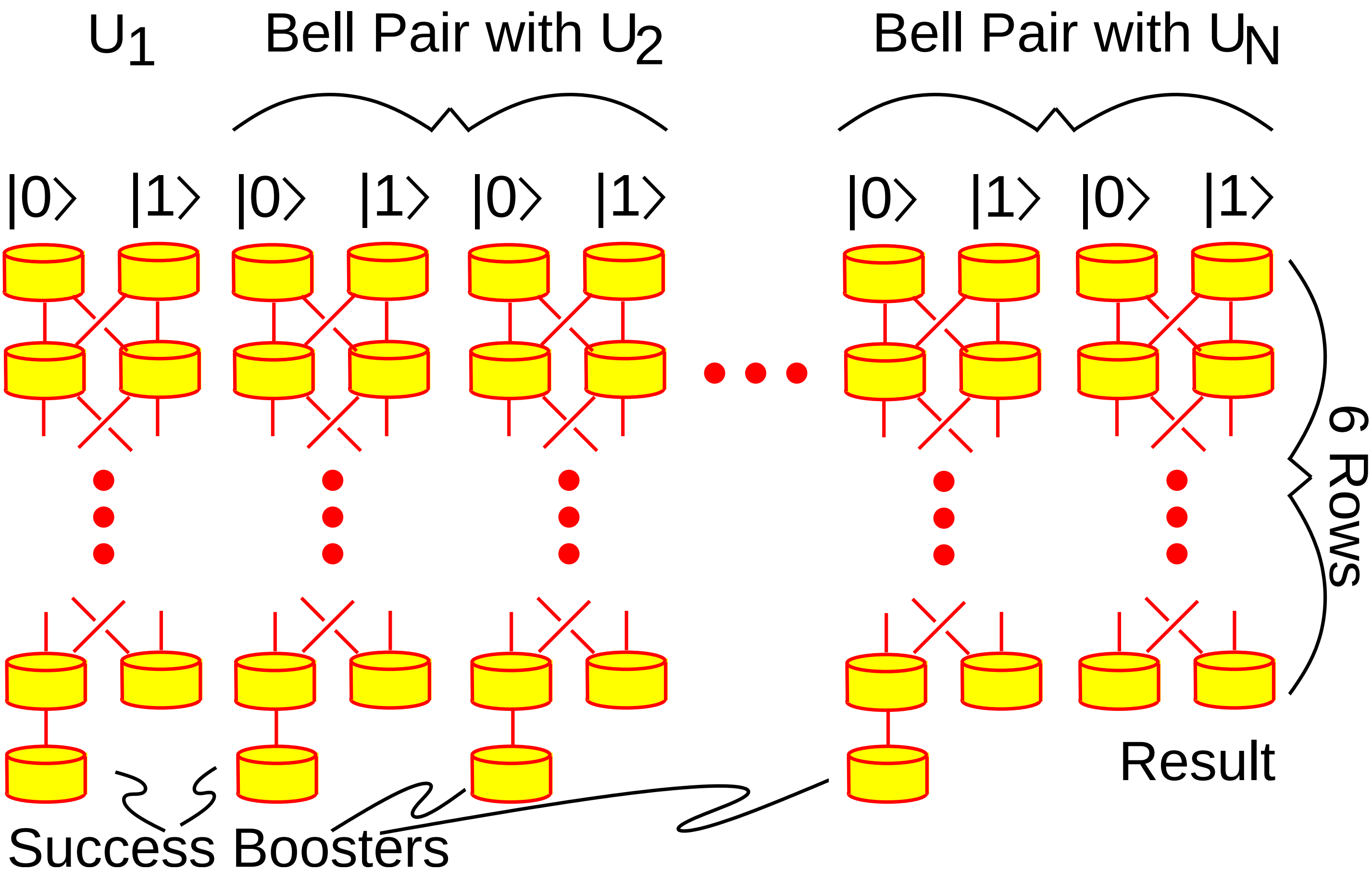
- Generalizing, we can apply gates $U_1, \ldots, U_N$ in parallel

$$U_1 |0\rangle \left(\frac{|0\rangle U_2 |0\rangle + |1\rangle U_2 |1\rangle}{\sqrt{2}} \right) \ldots \left(\frac{|0\rangle U_N |0\rangle + |1\rangle U_N |1\rangle}{\sqrt{2}} \right)$$

$$= \frac{1}{2^N} \left(\frac{|0\rangle |0\rangle + |1\rangle |1\rangle}{\sqrt{2}} \right) \ldots \left(\frac{|0\rangle |0\rangle + |1\rangle |1\rangle}{\sqrt{2}} \right) U_N \ldots U_2 U_1 |0\rangle$$
$$+ \text{ other terms}$$

- Parallel application of all gates succeeds with probability $(1/4)^N$
- The probability of success can be boosted arbitrarily close to unity within ground state quantum computation

Parallelizing ground state quantum computation



- Result $U_N \ldots U_2 U_1 |0\rangle$ obtained with $2N+1$ qubits and 6 rows
- Boosters cause probability to be $O(1)$ but make energy gap shrink to $O(1/N)$

Parallelizing ground state quantum computation

- Quantum gate teleportation also works with two qubit gates
- Generalization to arbitrary quantum computation
- Other algorithms using non-unitary development are conceivable

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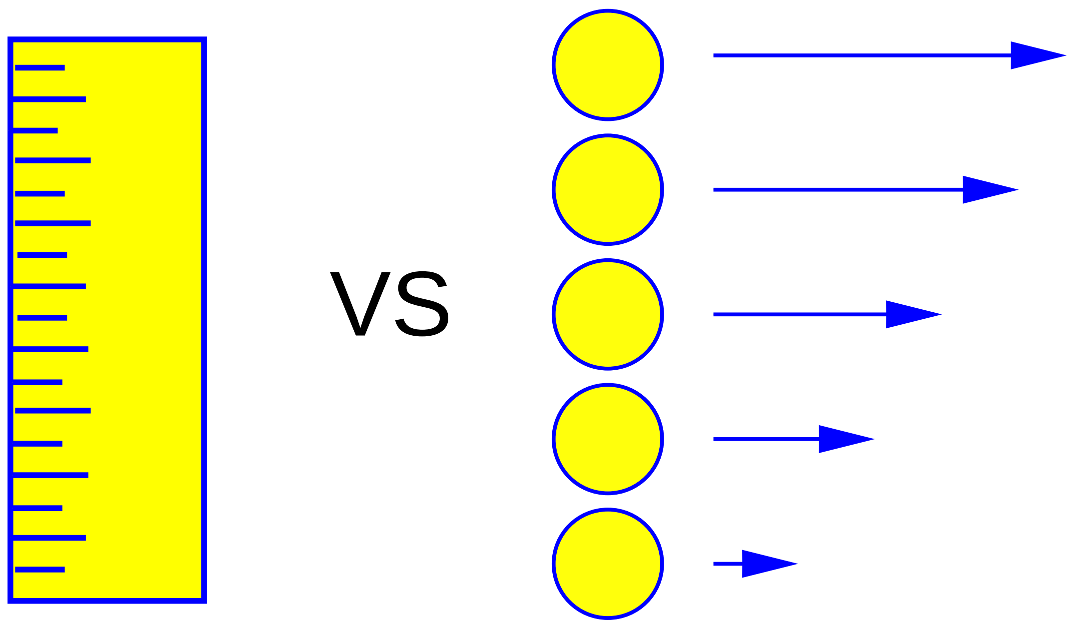
Future prospects

- Although we have focused on a quantum dot implementation, formalism is general
- For example, the qubits could develop in momentum space rather than real space



- Can we adapt the creative time-dependent quantum computation implementations?
- Josephson junction arrays may provide a realistic implementation
- How about clock cycles?

Summary

- We have proposed a ground state approach to quantum computation

- The Hamiltonian is straightforward, “modular”, and involves only one-body and two-body terms

- Energy gap $O(1/N^2)$ to decoherence

- Teleportation increases gap to $O(1/N)$ (stay tuned)

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