ARITHMETIC AND GEOMETRY OF ALGEBRAIC VARIETIES

WITH SPECIAL EMPHASIS ON

CALABI–YAU VARIETIES AND MIRROR SYMMETRY

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ABSTRACTS

Vincent Bouchard (Harvard University, Physics)

Mirror symmetry, matrix models and enumerative geometry

Recently we proposed a new, complete formalism, inspired by matrix models, to compute B-model open and closed topological string amplitudes in local Calabi-Yau geometries, including the mirrors of toric Calabi-Yau threefolds. The formalism is non-perturbative in the moduli, hence can be used to study various phases in the open/closed moduli space, such as orbifold points. In this talk I would like to summarize our B-model formalism, and focus on some of its mathematical implications, leading to new ideas/conjectures in mirror symmetry and enumerative geometry.

Chris Brav (Queen's University)

Braid groups and the McKay correspondence

We show how the Mckay correspondence for $\mathbf{P}^1$ gives rise to a braid group action on a category of equivariant sheaves on the cotangent bundle of $\mathbf{P}^1$ and then use this braid group action to relate different stability conditions on the cotangent bundle. The results are analogous to those of Tom Bridgeland on stability conditions for certain Fano varieties and their canonical bundles, but for more general types of braid groups.

Ethan Cotterill (Queen's University)

Rational curves of degree 11 on a general quintic threefold

We prove the "strong form" of the Clemens conjecture in degree 11. Namely, on a general quintic threefold F in $\mathbf{P}^4$, there are only finitely many smooth rational curves of degree 11, and each curve C is embedded in F with normal bundle $\mathcal{O}(-1) \oplus \mathcal{O}(-1)$. Moreover, in degree 11, there are no singular, reduced, and irreducible rational curves, nor any reduced, reducible, and connected curves with rational components on F.

Amanda Folsom (University of Wisconsin)

Harmonic Maass forms and Borcherds products

Abstract: Recent celebrated works of Zwegers and Bringmann-Ono have placed the mock Θ-functions and their generalizations in the context of weight $\frac{1}{2}$ harmonic weak Maass forms. In light of this, one expects similar correspondences to hold between other spaces of half-integral weight Maass forms, however missing are natural candidates to serve as analogues to the mock Θ-functions. In separate works with Bringmann-Ono and Bruinier-Bringmann-Ono, we make such correspondences precise by constructing half-integral weight vector valued harmonic weak Maass forms on the full modular group $SL_2(\mathbf{Z})$ whose transformation properties are dictated by the Weil representation arising from elementary theta series. We show that these vector valued Maass forms give rise to certain families of Borcherds products and also hypergeometric series. We establish correspondences between spaces of half-integral weight Maass forms and classical spaces of half-integral weight modular forms.

Alice Garbagnati (University of Milano)

Symplectic automorphisms on $K3$ surfaces

An automorphism σ of finite order of a K3 surfaces X is symplectic if and only if the quotient X/σ is again a K3 surface. Nikulin classified the finite abelian group acting symplectically on K3 surfaces and proved that the isometries induced by symplectic automorphisms on the second cohomology group of the K3 surfaces are essentially unique. We will describe these isometries by analyzing K3 surfaces with elliptic fibrations and symplectic automorphisms related to the geometry of these elliptic fibrations.

James Lewis (University of Alberta)

Cycles on Varieties Over Subfields of the Complex Numbers

In the context of algebraic and more generally cubic equivalence, we arrive at infinite rank results pertaining to spaces of algebraic cycles over certain subfields of the complex numbers, refining the recent works of Griffiths-Green-Paranjape and M. Saito.

Ling Long (Iowa State University)

Modularity of algebraic varieties

Abstract: Given a smooth irreducible algebraic variety X, we will discuss two different kinds of modularity concepts and their applications: 1) X is a fiber space over a modular curve of a finite index subgroups of the modular group and 2) certain l-adic Galois representations constructed from X are isomorphic to Galois representations arising from automorphic forms.

Michael Rose (University of British Columbia)

Mirror symmetry and l-adic Chen-Ruan cohomology

The Weil conjectures provide a technique to translate certain classical mirror theorems into the context of arithmetic algebraic geometry. I will demonstrate this strategy and give a survey of results in this direction.

Yifan Yang (National Chiao Tung University/Queen's University)

Monodromy and Sp_4 modularity of Picard-Fuchs differential equations for Calabi-Yau threefolds

In this talk we will first review the results in a recent joint work with Yao-Han Chen and Noriko Yui on the monodromy of Picard-Fuchs differential equations for Calabi-Yau threefolds. We will then propose an Sp_4-modular interpretation of these differential equations. The latter part is a preliminary report on a joint work with Wadim Zudilin.

Jeng-Daw Yu (Queen's University)

Unit roots of Calabi-Yau varieties in the Dwork families

We study the variation of the unit root along the Dwork families of Calabi-Yau varieties over a finite field by the method of Dwork-Katz and also from the point of view of formal group laws. A p-adic analytic formula for the unit roots away from the Hasse locus is obtained.

Yuri Zarhin (Pennsylvania State University)

Cubic surfaces and cubic threefolds, jacobians and intermediate jacobians

We discuss principally polarized complex g-dimensional abelian varieties that admit an automorphism of order 3 with only finitely many fixed points. It turns out that certain natural conditions on the multiplicities a and b of its action on the differentials of the first kind imply that those polarized varieties are *not* jacobians of curves. (For example, if $g = 5, a = 4, b = 1$ then the corresponding principally polarized abelian fivefold is *not* a jacobian.) The proof is based on the holomorphic Lefschetz–Atiyah—Bott fixed point formula.

As an application, we get another proof of the already known (thanks to Clemens and Griffiths) fact that intermediate jacobians of certain cubic threefolds are not jacobians of curves (and therefore those unirational threefolds are not rational varieties).