Robert Pollack, Boston University,
Iwasawa theory of elliptic curves

The main conjecture of Iwasawa theory for elliptic curves gives a precise relationship between a Selmer group (an algebraic object) and a p-adic L-function (an analytic object). The Selmer group contains information about the Mordell-Weil group and the Tate-Shafarevich group of E, while the p-adic L-function encodes the special values of twists of the complex L-series of E. Remarkably, through the theory of modular symbols, one can compute approximations to the p-adic L-function and thus (via the main conjecture) deduce arithmetic information about E -- for instance, information about the rank of the curve along the cyclotomic Z_p-extension and about the growth of Tate-Shafarevich group along this extension.

In this project, students will use modular symbols (and a computer!) to compute Mazur-Tate elements (which are the analogue of Stickelberger elements) for many different elliptic curves. From these Mazur-Tate elements, one can gain approximations to p-adic L-functions, compute \mu and \lambda-invariants, and determine bounds on ranks. Moreover, by considering ordinary or supersingular reduction -- good, multiplicative or additive reduction, one can see great differences in Iwasawa theory between varying reduction types. The aim is for students to discover many phenomenon of Iwasawa theory through explicit examples.