

Workshop on Noncommutative Dynamics and Applications
The Fields Institute, July 16-20, 2007

List of Confirmed Speakers

1. **L. Accardi** - Università di Roma “Tor Vergata” - accardi@volterra.mat.uniroma2.it
- **60 minutes**

- **Quantum Markovianity: a survey**

- **Abstract:** In the past 30 years the theory of quantum Markov chains (QMC) has undergone several developments.

The attempt to give an intrinsic operator–theoretical characterization of QMC produced a deep analysis, due to C. Cecchini, of various notions of quantum Markovianity.

The notion of Markovianity on CAR algebras, and the corresponding structure theorems, revealed a surprisingly richer structure than in the infinite tensor product case.

Moreover, as it often happens in quantum probability, the efforts to better understand the quantum case has lead to question some deeply rooted beliefs concerning classical Markov processes.

Applications to physics have proliferated in several different directions, ranging from the interesting theoretical results of Fannes, Nachtergale, Werner, Matsui, Mohari, Mukhammedov, Ohno,... to numerical simulations related to the Bethe approximation.

The results of Lindblad, Alicki and Fannes on the notion of quantum dynamical entropy and the subsequent extension by Ohya, Watanabe and others, have established a connection between QMC and the theory of quantum chaos.

Finally Petz and his school has shown that QMC play a relevant role also in quantum information, notably in the problems related to the capacity of quantum channels.

Now the boundary of this line of research is the extension of the above results to Markov fields (i.e. processes with multidimensional index set).

What is needed is not so much an abstract theory (many variants are possible and some of them already published) as a new non-trivial, class of concrete examples which could play for fields a role analogue to that, played in the 1–dimensional case, by the QMC, i.e. a benchmark on which to test the power of different theoretical proposals.

Now such class of examples has begun to be developed. I will use this class to illustrate some points of the abstract theory and some interesting open problems.

2. **A. Alevras** - US Naval Academy - alevras@usna.edu - **30 minutes**

- **Order and Equivalence of Endomorphism Semigroups**

- **Abstract:** An endomorphism semigroup α is greater than an endomorphism semigroup β if β is cocycle conjugate to a subordinate of α . Using results of R.T. Powers, G.L. Price and the speaker on the computation of the gauge group, we will present a proof that, in the case of endomorphism semigroups arising from one-dimensional boundary weights, this relation is antisymmetric: if α is greater than β and β is greater than α then α and β are cocycle conjugate.

3. **M. Anshelevich** - Texas A&M - manshel@math.tamu.edu - **60 minutes**

- **Free Meixner semigroups**

- **Abstract:** Free Meixner convolution semigroups are a class of free convolution semigroups of states on the algebra of non-commutative polynomials. They also have a more explicit representation as states on operator algebras. One of the properties shared by these semigroups is that the corresponding families of orthogonal polynomials are martingales. Examples of free Meixner semigroups include certain free product states, the free multinomial distribution, and its generalizations. All of these are, or may be, the free analogs of the quadratic exponential families in statistics.

4. **W. Arveson** - UC Berkeley - arveson@math.berkeley.edu - **60 minutes**

- **The noncommutative Choquet boundary**

- **Abstract:** Let S be an operator system – a self-adjoint linear subspace of a unital C^* -algebra A such that contains the unit of A and $A = C^*(S)$ is generated by S . A boundary representation for S is an irreducible representation π of $C^*(S)$ on a Hilbert space with the property that $\pi|_S$ has a unique completely positive extension to $C^*(S)$. The set ∂_S of all (unitary equivalence classes of) boundary representations is the noncommutative counterpart of the Choquet boundary of a function system $S \subseteq C(X)$ that separates points of X .

It is known that the closure of the Choquet boundary of a function system S is the Silov boundary of X relative to S . The corresponding noncommutative problem of whether every operator system has “sufficiently many” boundary representations was formulated in 1969, but has remained unsolved despite progress on related issues. In particular, it was unknown if ∂_S is nonempty for generic S . We show that every separable operator system has sufficiently many boundary representations. Our methods use separability in an essential way.

5. **B. Brenken** - University of Calgary - bbrenken@math.ucalgary.ca- **30 minutes**

- **Topological Quivers as Multiplicity Free Relations**

- **Abstract:** For a given topological quiver G we define an associated multiplicity free topological relation G^1 . We show that the unaugmented Cuntz-Pimsner C^* -algebras of G and G^1 are isomorphic if the range of the quiver G satisfies a finiteness condition and if the set of sinks of G is closed. There are counter-examples if either condition fails.
6. **N. Brown** - Penn State University - nbrown@math.psu.edu - **60 minutes**
- **Toward the C^* -classification of classical dynamical systems**
 - **Abstract:** I will discuss the classification of C^* -algebras associated to classical dynamical systems, both reviewing some history and looking at future prospects.
7. **F. Cipriani** - Politecnico di Milano - fabcip@mate.polimi.it - **60 minutes**
- **Differential Calculus and Fredholm Modules on Dirichlet spaces**
 - **Abstract:** The aim of the lectures is to show how any dynamical semigroup on a von Neumann algebra, symmetric w.r.t. a trace, gives rise to a closed derivation with values in a tangent bi-modulus. The derivation is a differential square root of the Dirichlet form associated to the L^2 -generator, which in turn can be represented as a generalized laplacian, i.e. as the divergence of a derivation. Example will include negative definite functions on groups as well as the Dirac laplacian on riemannian manifold. In the second part of the talk we will construct a summable Fredholm module associated to the Dirichlet form. As a distinguished situation we will consider Fredholm modules constructed starting from diffusions on fractals.
8. **D. Courtney** - UC Berkeley - djc@math.berkeley.edu - **30 minutes**
- **Lifting endomorphisms to automorphisms**
 - **Abstract:** Associated to any normal unital completely positive map (“UCP map”) on a von Neumann algebra is an abstractly defined W^* -dynamical system, its “asymptotic lift.” This generalizes the passage from an injective $*$ -endomorphism $\alpha : M \rightarrow M$ to its restriction to an automorphism of its “tail algebra” $\bigcap_{k \geq 0} \alpha^k(M)$. We explicitly calculate the asymptotic lift of an arbitrary endomorphism, and show how this computes the asymptotic lift of any UCP map in terms of its dilation theory. This is joint work with W. Arveson.
9. **K. Davidson** - University of Waterloo - krdaids@uwaterloo.ca - **60 minutes**
- **Operator algebras for multivariable dynamics**

- **Abstract:** (This is joint work with Elias Katsoulis.) Let X be a compact Hausdorff space with n continuous maps of X into itself. To this we associate various topological conjugacy algebras; and two emerge as the natural candidates for the universal algebra of the system. The classification of these algebras up to isomorphism leads to a new notion which we call piecewise conjugacy. Generalized notions of wandering sets and recursion are used to characterize when these algebras are semisimple.
10. **S. Dey** - Ernst-Moritz-Arndt-Universitat - dey@uni-greifswald.de - **30 minutes**
- **Constrained Liftings**
 - **Abstract:** (joint work with R. Gohm) Constrained dilations in context finite dimensional coisometric tuples of operators and its relation with minimal isometric dilation will be discussed. We show that a multi-analytic operator called constrained characteristic function is a complete invariant for constrained liftings. We obtain that some well-studied invariants of Hilbert modules are related to this characteristic function.
11. **D. Dutkay** - University of Central Florida - ddutkay@mail.ucf.edu - **30 minutes**
- **Covariant representations in wavelet theory**
 - **Abstract:** Motivated by wavelet theory we construct certain covariant representations associated to discrete dynamical systems. The spectral analysis of these representations reveals connections to random walks on branches, eigenvalue problems for transfer operators and orthogonality relations in wavelet theory.
12. **D. Evans** - Cardiff University - EvansDE@cf.ac.uk - **60 minutes**
- **Modular Invariants**
13. **R. Exel** - Federal University of Santa Catarina, Florianópolis - exel@mtm.ufsc.br - **60 minutes**
- **A new look at the Crossed-Product of a C^* -algebra by a Semigroup of Endomorphisms**
 - **Abstract:** We will describe a new notion of the crossed product of a C^* -algebra by the action of a semigroup of endomorphisms. This notion extends our earlier notion of crossed product by a single endomorphism for which one has chosen a transfer operator. We will give examples to illustrate the construction, some of which may also be described as groupoid C^* -algebras. Time permitting we will also show a curious counter-example based on the theory of cellular automata.

14. **R. Gohm** - University of Reading - r.gohm@reading.ac.uk - **60 minutes**
- **Characteristic Functions of Liftings and Applications to Completely Positive Maps**
 - **Abstract:** We introduce characteristic functions for certain contractive liftings of row contractions. These are multi-analytic operators which classify the liftings up to unitary equivalence and provide a kind of functional model. The most important cases are subisometric and coisometric liftings. We also identify the most general setting which we call reduced liftings. We derive properties of these new characteristic functions and discuss the relation to Popescu's definition for completely non-coisometric row contractions. Finally we apply our theory to completely positive maps and prove a one-to-one correspondence between the fixed point sets of completely positive maps related to each other by a subisometric lifting.
15. **D. Goswami** - Indian Statistical Institute, Kolkata -goswamid@isical.ac.in - **30 minutes**
- **Quantum stochastic dilations**
16. **I. Hirshberg** - Ben Gurion University of the Negev - ilan@math.bgu.ac.il - **60 minutes**
- **Permanence properties of strongly self-absorbing C^* -algebras**
 - **Abstract:** A strongly self-absorbing C^* -algebra is a unital, separable infinite dimensional C^* -algebra D satisfying that the first coordinate embedding from D into $D \otimes D$ is approximately unitarily equivalent to an isomorphism. A C^* -algebra A is said to be D -absorbing if $A \cong A \otimes D$. In this talk, I will survey some results concerning permanence of D -absorption under formation of crossed products and continuous fields. This is joint work with M. Rordam and W. Winter.
17. **M. Izumi** - Kyoto University - izumi@kusm.kyoto-u.ac.jp - **60 minutes**
- **Type III factors distinguish type III E_0 -semigroups.**
 - **Abstract:** In this talk I first present my recent work on a perturbation problem of the shift semigroup of $L^2(0, \infty)$. Then I discuss its application to E_0 -semigroups, which is a joint work with R. Srinivasan. I show that there exist uncountably many mutually non-cocycle conjugate type III E_0 -semigroups with the same Tsirelson invariant as that of CCR flow of index 1. These examples are distinguished by the type of the associated von Neumann algebras $A(U)$, where U is a bounded open subset of the half-line.
18. **P. Jorgensen** - University of Iowa, jorgen@math.uiowa.edu - **60 minutes**

- **Uses of operator algebraic methods in dynamics and in wavelet algorithms**
 - **Abstract:** The talk will give an overview of some recent research trends involving operator algebras and dynamics, with an emphasis on work in which I have myself been involved, together with co-authors Larry Baggett, Ola Bratteli, Berndt Brenken, Dorin Dutkay, Kathy Merrill, Judy Packer, Steen Pedersen, Myung-Sin Song; among others. The use of dynamics center around recursive iteration schemes as they are used for example in Iterated Function Systems (IFSs), a special class of fractals, in wavelet algorithms, and in such applied areas as signal and image processing. A leading operator algebraic theme will be a certain class of representations of the Cuntz algebras.
19. **R. Kadison** - University of Pennsylvania - kadison@math.upenn.edu - **60 minutes**
- **Means of Unitary Operators Revisited**
 - **Abstract:** This lecture will be about the ways in which operators in a self-adjoint operator algebra can be decomposed as convex combinations of unitary operators. The main result is a proof of a sharp inequality for the minimum number of unitary operators needed. (joint work with Uffe Haagerup and Gert Kjærgaard Pedersen - to appear in the volumes of Math. Scand. dedicated to the memory of Gert)
20. **C. Koestler** - Carleton University - koestler@math.carleton.ca - **60 minutes**
- **On endomorphisms of von Neumann algebras from braid group representations**
 - **Abstract:** Recently we have proven a noncommutative version of the extended De Finetti theorem for infinite sequences of random variables. These sequences canonically lead to endomorphisms of von Neumann algebras. In contrast to the classical result, the distributional symmetries of exchangeability and contractibility (aka "spreadability") are shown to be no longer equivalent in quantum probability. In joint work with R. Gohm, we have now identified an interesting class of contractable noncommutative random sequences. My talk will report some of our new results. We will show that braid group representations induce contractable noncommutative random sequences. We prove that these random sequences lead to triangular towers of von Neumann algebras, such that all cells form commuting squares. Our approach is applicable for nonhyperfinite von Neumann algebras and includes all examples from the Jones fundamental construction. We will illustrate our results by examples coming from the left regular representation of the braid group and the free group.
21. **D. Kribs** - University of Guelph - dkribs@uoguelph.ca - **60 minutes**

- **Some Mathematical Aspects of Quantum Error Correction**

- **Abstract:** Researchers in the field of quantum error correction (QEC) are primarily focussed on the development of approaches to overcome fundamental obstacles associated with physical implementations of quantum computing. QEC theory has evolved over the past decade from a collection of seminal examples and techniques, to a larger collection of seminal examples and techniques. In other words, much progress has been made, but there is still much to do. As the field has developed, it has also become apparent that there are links with other disciplines in quantum information science, such as quantum cryptography and quantum gravity.

I'll begin this talk by reviewing the basic mathematical framework for QEC. Then I'll describe some of my recent work that has been motivated by (and hopefully will soon motivate) experimental efforts in quantum computing. If time permits, I'll also discuss some of the connections mentioned above.

22. **M. Laca** - University of Victoria - laca@math.uvic.ca - **60 minutes**

- **Symmetries and equilibrium in C^* -dynamical systems.**

- **Abstract:** By examining several concrete C^* -dynamical systems (based on Cuntz-Krieger algebras, Hecke algebras, and the recent GL_2 system of Connes-Marcolli, among others) I will illustrate how the stability implicit in the KMS condition brings about symmetries and rescaling properties that can be used to describe the equilibrium state spaces in terms of measures.

23. **M. Lindsay** - University of Lancaster - j.m.lindsay@lancaster.ac.uk - **60 minutes**

- **Quantum dynamical semigroups: duality and generation**

- **Abstract:** A Markov semigroup which is symmetric with respect to a measure m on the underlying state space is, on the one hand a selfadjoint contraction semigroup on the Hilbert space $L^2(m)$ which therefore has a (closed densely defined) quadratic form generator, and on the other hand is positive and contractive on the algebra of essentially bounded measurable functions. The latter properties are encoded at the L^2 -level by the Beurling-Deny conditions: the form reduces under so-called normal contractions, in short it is a Dirichlet form. This characterization has a satisfactory extension to non-symmetric semigroups. A noncommutative theory requires a proper analogue of both 'symmetry/adjoint', now with respect to a state or weight on a von Neumann algebra, and 'Dirichlet'. In this talk the notion of 'KMS-adjoint' will be contrasted with the more naive notion of 'GNS-adjoint' and it will be shown how the natural analogue of Beurling-Deny conditions still characterize the form generators of Markov semigroups in the noncommutative domain.

24. **R. Longo** - Università di Roma “Tor Vergata” - longo@mat.uniroma2.it
- **60 minutes**
- **Real Hilbert Spaces, $SL(2, \mathbb{R})$, Modular Theory and CFT**
25. **D. Markiewicz** - Technion—Israel Institute of Technology - dmarkiew@tx.technion.ac.il
- **30 minutes**
- **The Gauge Group of a Strongly Spatial E_0 -semigroup**
 - **Abstract:** (Joint work with Robert T. Powers). The group of local unitary cocycles of an E_0 -semigroup, also called the gauge group, acts in a natural way on the set of units of the E_0 -semigroup. It follows from work of William Arveson that for a completely spatial E_0 -semigroup of index n this action is not only transitive, but also $n+1$ -fold transitive: the group acts transitively on the set of normalized $n+1$ -tuples of independent units. In this talk we will analyze the local unitary cocycles of a class of spatial E_0 -semigroups which we call strongly spatial. As an application we will present examples of strongly spatial E_0 -semigroups of index 1 whose gauge groups are not 2-fold transitive.
26. **H. Osaka** - Ritsumeikan University - osaka@se.ritsumei.ac.jp - **60 minutes**
- **Inclusion of C^* -algebras**
 - **Abstract:** We discuss three basic properties for C^* -algebras, Property (SP), topological stable rank, and cancellation property.
- Let $1 \in A \subset B$ be an inclusion of C^* -algebras with common unit and $E: B \rightarrow A$ be a faithful conditional expectation with index finite type. Assume that A is simple and has Property (SP). Then we have
- (a) B has Property (SP).
 - (b) $\text{tsr}(B) \leq 2$ if $B \subset A$ is of depth 2 and $\text{tsr}(A) = 1$.
- As an application
- if $B \subset A$ is of finite depth and A has cancellation property with $\text{tsr}(A) = 1$, then B has cancellation.
27. **R. Powers** - University of Pennsylvania - rpowers@math.upenn.edu - **60 minutes**
- **Comparison theory for E_0 -semigroups**
 - **Abstract:** We discuss comparison theory for E_0 -semigroups. The first step is defining the notion of cocycle conjugacy for E -semigroups (Strongly continuous semigroups of $*$ -automorphisms of $B(H)$ which are not necessarily unital). Using Arveson’s Theory of product systems one can show every E -semigroup of $B(H)$ is cocycle conjugate to

an E_0 -semigroup. We say one E -semigroup is greater than a second E -semigroup if it is cocycle conjugate to a subordinate of the second E -semigroup. The properties of this ordering as well as questions that arise are discussed. The ordering helps clarify how the notion of index fits in with the classification of E -semigroups of $B(H)$. In particular it appears that the index of an E_0 -semigroup of type III should be minus infinity.

28. **G. Price** - US Naval Academy - glp@usna.edu - **60 minutes**

- **On some E_0 -semigroups induced from CP -flows**
- **Abstract:** Robert Powers has shown that all spatial E_0 -semigroups can be induced, up to cocycle conjugacy, from completely positive semigroups known as CP -flows. We use the construction of E_0 -semigroups from CP -flows to discuss how in some respects the comparison theory for type II E_0 -semigroups can behave like the comparison theory for projections in a type II factor.

29. **I. Putnam** - University of Victoria - putnam@math.uvic.ca - **60 minutes**

- **C^* -algebras from hyperbolic dynamical systems**
- **Abstract:** A Smale space, as defined by David Ruelle, is a topological dynamical system having a type of hyperbolic structure. Examples include Anosov diffeomorphisms, certain solenoids, certain substitution tiling systems, shifts of finite type and basic sets from Smale's axiom A system. It is possible to construct C^* -algebras from such systems. In the case of shifts of finite type, these are AF-algebras. The talk will describe the basic ideas of Smale spaces, along with examples, the construction of the C^* -algebras and various properties which they possess. In particular, there is a K-theoretic duality between a pair of them. Finally, the talk will describe a homology theory for Smale spaces which is closely related to the K-theory of the C^* -algebras.

30. **O. Shalit** - Technion - orrms@tx.technion.ac.il - **30 minutes**

- **E_0 -dilation of a pair of strongly commuting CP_0 semigroups**
- **Abstract:** Given a pair a commuting CP -semigroups Θ and Φ acting on von Neumann algebra M in $B(H)$, we are interested in constructing a pair of commuting E -semigroups α and β , acting on some larger von Neumann algebra R in $B(K)$, where K contains H , such that for all $s, t > 0$ and all T in R , $\Theta_s(\Phi_t(P_H T P_H)) = P_H \alpha_s(\beta_t(T)) P_H$. In other words, we are interested in constructing in E -dilation to a two-parameter CP -semigroup. In this talk we will show that if Θ and Φ are unital and strongly commuting, then an E_0 -dilation exists (the definition of strong commutation is technical, and will be explained in the talk. For now it will be enough to say that there

are many pairs of strongly commuting CP -semigroups). We will also discuss our progress in the construction of an E_0 -dilation to a k -tuple of strongly commuting CP_0 -semigroups.

This work is part of the author's PhD. thesis, done under the supervision of Baruch Solel.

31. **B. Solel** - Technion—Israel Institute of Technology - mabaruch@technix.technion.ac.il
- 60 minutes

- **Operator Algebras Associated with Unitary Commutation Relations**

- **Abstract:** This is a joint work with S.C. Power. We define non-selfadjoint operator algebras with generators $L_{e_1}, \dots, L_{e_n}, L_{f_1}, \dots, L_{f_m}$ subject to commutation relations given by a unitary $mn \times mn$ matrix u . These algebras, which generalize the analytic Toeplitz algebras of rank 2 graphs with a single vertex, are classified up to isometric isomorphism in terms of the unitary matrix u .

32. **K. Sinha** - Indian Statistical Institute, Kolkata - kbs_jaya@yahoo.co.in - 60 minutes

- **Quantum Dynamical Semigroups and their Stochastic Dilations**

- **Abstract:** A theory is developed the stochastic dilation for a class of Quantum Dynamical Semigroups (QDS) on a $*$ -algebra which admits an action of a Lie group such that the QDS is covariant wrt the action. Several applications including that for the Kronecker foliation of a 2-torus are studied.

33. **M. Skeide** - Università degli Studi del Molise - skeide@unimol.it -60 minutes

- **Products Systems and Quantum Dynamics**

- **Abstract:** Quantum Dynamics, both reversible and irreversible, gives rise to product systems. Arveson's discovery that reversible dynamics on $B(H)$, the algebra of all operators on a Hilbert space H , leads to product systems of Hilbert spaces, marked the begin of a still increasing amount of literature where product systems are used in order to understand better the quantum dynamics that produced them. This intimate relationship between quantum dynamics and product systems has been generalized in many ways, often leading to product systems of Hilbert modules.

For product systems of Hilbert spaces it is impossible to give an approximately complete description of the known constructions in just one talk. And still many questions (in particular about classification) are open. The scope of our talk is to make an attempt to present a picture of the state of the art regarding product systems of Hilbert

modules. This may appear as a contradiction. But as we intend to put emphasis on those aspects that distinguish the module case from the scalar case, and as the theory in the module case is less developed, we hope the attempt might be successful.

34. **R. Srinivasan** - Chennai Mathematical Institute - vasanth@cmi.ac.in - **30 minutes**

- **Title: TBA**

35. **B. Tsirelson** - Tel Aviv University - tsirel@post.tau.ac.il - **60 minutes**

- **Automorphisms of the type II Arveson system of Warren's noise**
- **Abstract:** Motions of the plane (shifts and rotations) correspond to automorphisms of the type I Arveson system of white noise. I prove that automorphisms corresponding to rotations cannot be extended to the type II Arveson system of Warren's noise.