General Approaches for Modelling Liquidity Effects in Asset Markets and their Application to Risk Management Systems

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Overview

1. Role of Risk Management Systems in Financial Institutions.
2. RM Theory - Applied Arrow-Debreu asset models, dynamic factor models, arbitrage pricing.
3. Illiquidity - Some modeling suggestions for RM systems.
   (a) Illiquidity as transaction costs for assets.
   (b) Illiquidity as funding constraints, margin, VaR etc.
   (c) Illiquidity as price pressure effects on trading.
4. Systemic risks
5. Early days: unsolved problems and puzzles.
6. Some suggestions

A Quick Primer on RM modelling

Recall the basic two date model:

$$\max_{a_k} U_f(x_1)$$

such that

$$x_1 \leq \sum_k R_{1k} a_k - C_1 + K_1$$

$$W_0 = \sum_k p_k a_k$$

where K_1 is capital and C_1 are total stochastic commitments. The objective may not be well defined in incomplete markets. If markets are complete, then $U_f(x_{1f}) = E_Q[x_{1f}]$ where Q is the martingale measure or Arrow Debreu price vector or measure.

Modifications

Factor Structure on returns:

$$R_{1k} = \sum_f F_f \beta_{fk} + \epsilon_k \text{ for all } k = 1, \ldots K \tag{1}$$

Model accommodates equity positions, govt. bonds, corporate bonds with default, derivatives.

Implications:

a. Use of diversification across independent ϵ_f's, leads to APT.
b. Common factors allow arbitrage pricing for derivatives, hedge fund strategies etc.
c. This model is the basis for credit derivatives and their off-spring.

Arrow-Debreu:

If the model is embedded in closed, competitive market clearing system, asset markets are complete, then the allocations are efficient. Moreover,

- The model cannot have bubble or crashes.
- Value maximization is well-defined.
- Accounting is trivial. All valuations are available as mark to market.
- All assets are traded on perfectly liquid markets.
- Systemic network effects are solved as part of the competitive equilibrium. Bankruptcy chains are all rationally anticipated. (See Milne (1976)).
- There is no rationale for regulation and banking structures.

These observations are discussed recently by Gromb and Vayanos (March 2010) -and others. They discuss limits to arbitrage and empirical examples inconsistent with arbitrage pricing, crashes, etc.

Multi-period Version

Taking a finite tree or the continuous time counterpart, the model becomes

$$\max_x U(x) \text{ s.t.}$$
$$x = Ra - Pa - C + K$$

Where:

- x, C, K are stochastic cash flows
- R is the stochastic dividend, coupon etc stochastic process
- P is the stochastic price process and a is the predictable asset strategy

- Dynamic models of derivative pricing and hedging can be placed in this model.
- Dynamic Factor and APT models also special cases.
- General model allows contingent trading strategy.
- But the model allows far more interesting dynamic trading, hedging, arbitrage pricing etc.
- The objective with complete markets will be unique and linear as a multi-period extension of the AD price functional.

BUT

- Multi-period A-D model. All the damaging two date implications follow for the multi-period model.

Variations for Risk Management

RM does not attempt to solve the whole problem, but explores the constraint set for the one period ahead version, by assuming the current positions (a_k^*) and considers the implied return distribution of the one period ahead returns, x_1

$$x_1 = \sum_k R_{1k} a_k^* - C_1 + K_1 \tag{2}$$

Usually the RM also considers deviations Δ_k on the positions to test for sensitivity:

$$x_1 = \sum_k R_{1k}(a_k^* + \Delta_k) - C_1 + K_1 \tag{3}$$

More sophisticated versions look a small number of periods ahead, postulate strategies and simulate multi-period versions of deviations.

Note: The model is still basically AD with all its limitations. Playing with the joint probability distributions, econometric estimation, changing the stochastic processes etc. does not avoid the crucial misspecification that all markets are assumed liquid - always!

RM Models and Banking Theory: Mind the Gap

Modern Banking Theory is based on a more realistic theory allowing asymmetric information and strategic behaviour. (Game theory models that exploit ideas from Finance and Industrial Organization theory.)

- It emphasizes liquidity problems stemming from moral hazard, adverse selection, incomplete contracts, etc.

Basic derivative pricing and hedging, and RM theory based on A-D. None of the above frictions apply.

- A-D models are frictionless, banks earn zero profits as they are irrelevant under Modigliani-Miller.
- This is well-known in the theory - see Freixas and Rochet (2008). The related Allen and Gale (2007) book discusses models of illiquidity and financial crashes.

Private RM's were either unaware or vaguely aware of modern banking theory and systemic risk modelling - "What is systemic risk?" Now there are attempts to model these effects.

Most banking models have small dimensions - do not model RM seriously - a major gap in the theory (and practice) between RM and banking theory.

Market Microstructure models look carefully at frictions in trading, but are usually small dimension. (For an introduction see the monographs by O'Hara (1997) and the more recent Hasbrouck (2007). We will reference some of these models.

Market microstructure has made inroads in the most sophisticated FI's.

Macro-Prudential Systemic Risk Models

Theoretical models are usually small dimension - Allen and Gale, Rochet et al.

Empirical models constructed by Central Banks (e.g. Elsinger, Lehar and Summer (2005); RAMSI at the Bank of England) use simple model of banks, balance sheet data and algorithms. They model liquidity in a simple add-on manner. They test for bank failures and systemic failures. In the latest crisis, they failed to predict the mess in the UK (see Haldane (2009)).

These models are becoming more sophisticated, but still have major limitations in capturing investment bank exposures and more complex counterparty risks. (See comments by Upper (2007)).

Recent discussions of networks are a natural extension of this approach - see for example: Shin (2008), Haldane (2009) and Gai and Kapadia (2009).

The current network approach is limited in that most models do not introduce behavioural reactions by banks - but see Gai and Kapadia (2009) for a recent simple attempt.

Recent Network Game theory results may provide a way ahead. (Work in progress with Gai, Milne and Thompson.)

These network games need to introduce liquidity impacts from trading. Modified network models developed by Central Banks (e.g. Austrian and UK models) indicate that liquidity losses exacerbate the losses and contagion.

RM Systems and Liquidity Risks

The AD model - and the basic RM model - assumes no liquidity problems.

How can we exploit the richness of the RM models, and yet extend them to incorporate liquidity?

Keep it basic, and not too difficult. Better to be approximately right than exactly wrong.

Basic Approaches:

- Market Liquidity as transaction costs.
- Funding Liquidity and other Portfolio Constraints
- Liquidity as market power.
- Externalities related to liquidity - e.g. bank runs.

These approaches have been explored in the literature and can mimic key features of what is thought of as “liquidity.”

Models should be simple enough to use with the existing systems.

Liquidity as Transaction Costs

This is an old model dating from GE in the 1970's.

- It assumes buying and selling prices (bid-ask) generated by "transaction costs".
- This formulation doubles the size of the asset trading space (buying and selling of assets are treated as different assets trading through intermediaries with costs.
- The model has some nice features.

Its weakness is that it fakes the transcation costs as just "costs", no discussion of search, asymmetric information or strategic behaviour.

- It does not explain why TC in some markets are higher than in others.
- It assumes that buying and selling prices are competitive.
- General Asset Model with TC - equilibrium existence proofs well-known (e.g. Jin and Milne (1999)).
- Lengthy literature, partial characterizations, but still much more work required to make the results operational.

Quick Sketch of Characterization Literature

The basic agent model:

$$\max_{x_i \in X_i} U_i(x_i)$$

s.t.

$$x_i = R[\Delta_i^B - \Delta_i^S] + S\Delta_i^S - B\Delta_i^B + \bar{x}_i + y_i;$$

$$(\Delta_i^B, \Delta_i^S, y_i, S, B) \in T_i.$$

From the first order conditions, the personalized expectation of future payoffs must lie between the buying and selling price. (e.g. Jouini and Kallal (1995), and Milne (2010) for discrete time - state versions).

$$P_k^B(\omega_t) - \frac{\delta_{ki}^B(\omega_t)}{\lambda_i(\omega_t)} = \sum_{s>t} \gamma_i(s \mid \omega_t) \sum_{S(s|\omega_t)} R_k(\omega_s)\tilde{p}_i(\omega_k s \mid \omega_t) \quad (4)$$

$$= P_k^S(\omega_t) + \frac{\delta_{ki}^S(\omega_t)}{\lambda_i(\omega_t)}; \quad (5)$$

Defining $P_k^i(\omega_t) \equiv \sum_{s>t} \gamma_i(s\omega_t) \sum_{S(s|\omega_t)} R_k(\omega_s)\tilde{p}_i(\omega_s \mid \omega_t)$

$$\begin{aligned} P_k^B(\omega_t) &\geq \\ P_k^i(\omega_t) &= \gamma_i(t+1 \mid \omega_t) \sum_{S(t+1|\omega_t)} \{P_k^i(\omega_{t+1}) + R_k(\omega_{t+1})\}\tilde{p}_i(\omega_{t+1} \mid \omega_t) \\ &\geq P_k^S(\omega_t) \end{aligned}$$

Dynamic portfolio models with highly restrictive preferences, asset processes, constant TC. (e.g. Basak and Cuoco (1998); Dai, Jin and Liu (2008).)

The basic story is that trading is “sticky”around the initial asset endowment; that trading takes place as soon as subjectively expected returns lie outside the buy-sell price band. Otherwise no trades.

Portfolio model does allow flexibility in considering changes in TC over time to mirror periods of “illiquidity”.

This will imply that there will be a demand for “liquid”assets that may earn a smaller return but have lower transaction costs to liquidate if contingent liabilities come due.

Danielsson and Zigrand (2007) have a simple GE asset economy and show the second best flavour of these constrained equilibria and "surprising" welfare perversities.

But they are not surprising - they are standard variants of GE second best results.

Welfare perversities in incomplete markets, or other second best GE - an old story Hart (1975), but often forgotten, then rediscovered.

The Diamond Dybvig (1983) Bank Runs story can be seen as an example of this type of second best story - see Allen and Gale (2007) for a full discussion of this liquidity argument.

- Note: incomplete asset market economies can be modelled as TC economies with two classes of asset markets:
    1. No transaction costs;
    2. Other assets have with infinite TC.

- Observe that with wide bid-ask spreads then there is no unique valuation for an asset.
- Mark to market will depend on buy or sell strategies.
- Subjective and judgmental valuations appear in the FO conditions.
- This has immediate implications for accounting rules, the necessity for RM "judgment" and careful analysis of holding to maturity strategies.
- The model has insights, but cannot deal with the underlying causes of bid-ask spreads varying over time and states, particularly in times of crises.
- Some simple models with asymmetric information etc. are now appearing. For a partial survey see Tirole (2009).

Transaction Costs and Static Hedging

It is common for traders to not hedge dynamically - as is the case in Black-Scholes et al. They make static hedges using other derivatives to “save on transaction costs”. This type of behaviour is easy to model in our framework.

An easy example is a security that has the same terminal stochastic cash flow as a portfolio. Both the security (derivative) and other assets have transaction costs. Then it is easy show Milne (2010) that one can obtain pricing bounds on actively traded securities.

In particular, if transaction costs are proportional to prices, the asset markets are active, and the same proportion applies to all securities, then

$$P^B_{k'}(\omega_t) = \sum_{k \in F} \alpha_k(\omega_t) P^B_k(\omega_t) : \alpha_k(\omega_t) \geq 0$$

$$P^S_{k'}(\omega_t) = \sum_{k \in F} \alpha_k(\omega_t) P^S_k(\omega_t) : \alpha_k(\omega_t) \geq 0$$

If we take the midpoint of the bid-ask spread then we get a mid-point pricing result that mimics the standard pricing result.

A-D Asset Economies with Trading Constraints and Personal Transaction Costs

This economy is easy to specify see Milne (2010)). Assuming that $P^B = P^S$ so that general market TC is zero, the first order conditions are:

$$\frac{\delta^B_{k`i}(\omega_t)}{\lambda_i(\omega_t)} + \sum_{\ell} \frac{\delta_{i\ell}(\omega_t)}{\lambda_i(\omega_t)} \frac{\partial F_{i\ell}}{\partial \Delta^B_{ik}(\omega_t)} - P_k(\omega_t) + \sum_{s>t} \gamma_i(s \mid \omega_t) \sum_{S(s|\omega_t)} R_k(\omega_s)\tilde{p}_i(\omega_s \mid \omega_t) = 0. \quad (6)$$

$$\frac{\delta^S_{ki}(\omega_t)}{\lambda_i(\omega_t)} + \sum_{\ell} \frac{\delta_{i\ell}(\omega_t)}{\lambda_i(\omega_t)} \frac{\partial F_{i\ell}}{\partial \Delta^S_{ik}(\omega_t)} + P_k(\omega_t) - \sum_{s>t} \gamma_i(s \mid \omega_t) \sum_{S(s|\omega_t)} R_k(\omega_s)\tilde{p}_i(\omega_s \mid \omega_t) = 0. \quad (7)$$

Characterizations

- Characterizations are quite easy in the abstract. First order conditions will include constraint Kuhn-Tucker multipliers (Milne (2010) that distort the classic martingale pricing conditions.
- There are many simple GE versions of the model that assume restrictive preferences and a small sample of assets.
  - For a recent paper see Garleanu and Pedersen (2009) (and their bibliography) on pricing distortions induced by margin requirements.

- These and the related TC models can induce distorted martingale pricing conditions to obtain factor models with “liquidity”factors.
  - e.g. Acharya and Pedersen (2005) is a CAPM pricing model with additional factors generated by transaction constraints and costs.
- One variation of this type of model allows asset prices as part of the constraint set - then price movements can induce asset trading if the constraints are binding, precipitating further selling and price declines in a crisis cycle.

Dynamic Hedging and Pricing

Using the personal TC model, it is easy to adapt it obtain known results on dynamic hedging. Jarrow and co-authors have a series of papers obtaining pricing bounds for dynamic hedges when there are non-linear personal TC or price pressure effects (the model is basically the same).

Manipulating the first order conditions one can obtain, using recursion, price bounds for a European option. The intuition is simple:

- The costs for not obtaining a perfect hedge are traded off against the present value of the TC accumulated in the dynamic hedge.

Illiquidity as Oligopoly Pricing

Another intuitive measure of “illiquidity” is the idea that large trades can move prices. Markets are said to be “thin”.

For a long time, Finance theory and empirical work argued that this “price pressure” argument was negligible, markets were competitive, and price movements signalled information changes.

- It is a curious argument for financial economists to argue that all financial markets were competitive. In many financial markets the evidence is to the contrary. Ask any large trader.

More recent empirical work supports the economic intuition that large trades (relative to market holdings) will move prices.

The Market Microstructure literature addresses this type of problem - see O'Hara (1995).

- For a more recent review of simple strategic trading models see Hasbrouck (2007). The models are not strictly game theoretic, but are reduced forms models.

A Partial View: The RM system with (Market Power) Illiquidity

Consider a modified RM problem - where the FI trades can move prices.

$$\max V(x) \text{ s.t.} \tag{8}$$

$$x = R\Delta - P(\Delta)\Delta + R(a^*) - C + K \tag{9}$$

$P(\Delta)$ is the price stochastic process as a function of the trading strategy a^* is the asset endowment.

Analysis of this problem will depend on the specification of the price process function $P(\Delta)$. There are two basic processes used in this literature, temporary and permanent price effects.

Reduced Form Specification

- The price process has a temporary impact from a trade. This process provides incentives to spread out inter-temporal trades for an asset to minimize price impacts over time.
- Also in a portfolio, it will provide incentives to liquidate (other things equal) the assets that are most liquid - i.e. assets that have elastic demand curves.
- Dynamics can be tricky as there will be a trade-off between risk and liquidation strategy.
- Jarrow et al (as we observed above) and other writers in the derivative literature use this reduced form specification to provide price bounds on derivatives. Clearly derivative pricing is very sensitive to the trading strategy.
- Market microstructure and trading models use this type of model to “spread out” trades over time.

A Strategic View: The RM System with (Strategic) Illiquidity

The idea here is to consider prices impacted by the dynamic trades of several oligopoly traders, $h = 1, \ldots, H$.

The Model:

$$\max V_h(x) \text{ s.t.} \tag{10}$$

$$x_h = R\Delta_h - P(\Delta_h, \Delta_{-h})\Delta_h + R(a_h^*) - C_h + K_h \tag{11}$$

- Looks simple enough in the abstract, but in finite time requires subgame perfect techniques to solve.
- Also will require careful specification of the $P(\Delta_h, \Delta_{-h})$ function. Brunnermeier-Pedersen (2005) and others have specifications to deal with existence of equilibria, and possible multiple equilibria.
- Model, in principle, can handle many interesting strategic situations - predation, interbank pressures, predation as a prelude to takeover, possible roles for central bank intervention, etc.
- A fascinating and interesting research program, and highly relevant in the current crisis.

Early Days!

Lessons for RM?

This class of models is too tricky to use in RM - except in a very stripped down form. They require a view on competitor strategies, the price process function etc.

Way beyond most current RM capabilities.

But the ideas would inform FI strategies and simple partial fixes to the RM system to capture liquidity impacts of a strategy.

Non Price Externality Models

This class of models is an extension of the model with asset constraints but with an additional twist: other agent's actions appear in the constraints.

For example consider asset positions where other agents can impose margin, or call for asset liquidation. The classic example of this type of model is the Diamond and Dybvig (1983) and its successor models. The resulting model involves a coordination game where there can be multiple equilibria if depositors may or may not withdraw beyond the limits of liquid asset holdings.

Other examples are informational herding models that move asset prices.

These types of models are not adequately modelled in the RM systems.

Systemic Risks, Liquidity Issues and Regulation

We have the central bank macro models of systemic risks - but currently they do not take the RM systems, and liquidity modelling seriously.

Occasional stress tests by the regulators of individual banks RM systems. Also occasional stress tests of all banks in a system to a shock, but limited feedback effects.

Can we exploit the FI's RM systems, and strategic decisions?

Suggestion: Financial War Games?

Idea to exploit the FI's RM systems, to check for system weaknesses. Coordinated by the Financial Regulator

Steps:

1. Impose a hypothetical event shock to all the FI's, check through the RM system for the first round impact.
2. Require the FI's to reveal their position responses to the regulator.
3. The regulator uses residual demand curves to simulate "illiquidity" in certain markets.
4. Check counterparty risks across regulated FI's.
5. Iterate - small number of iterations should produce obvious weaknesses.

Weaknesses

Many weaknesses!

- FI's will be wary of revealing strategies.
- Games will be only as revealing as the design permits and the input of the FI's. If some FI's are perfunctory, then this reduces the usefulness for the system analysis.
- If repeated, penalties could be imposed for not performing.
- Regulators must be up to speed as well - if they do poor design, FI's will lose interest fast.

Strengths

- Educative - forces RM's and FI strategists to think through scenarios. Can be used to think through the implications of liquidity crises and CB responses.
- Educative for the regulator. Feedback may reveal weakness in private bank RM systems, system reactions, etc.
- Educative for central banks in seeing the impact on strategies in providing different liquidity provisions. They can think through bank reactions to various liquidity support measures.
- If taken seriously FI's may well play strategically to hide real responses from competitors - the regulator can test to see if they react in real situations consistent with the game responses.