The Evaluation of Swing Contracts with Regime Switching

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Plan of Talk

- Basic Swing Contracts with Make-up and Carry forward provisions
- Forward Price Curve with Regime Switching Volatility
- Setting up the Optimisation Problem
- Pentanomial Tree Approach
- Numerical Examples
- Conclusion

1 Literature Review

- Theoretical: Carmona and Touzi [2008] develop a mathematical framework for swing options viewed as nested optimal-stopping problems.
- Binomial and Trinomial Trees: Thompson [1995], Clewlow, Strickland, and Kaminski [2001a,b] describe features and valuation approach of single year swing contract using trinomial tree approach.
- Simulation: Ibáñez [2004] seeks to determine an approximate optimal strategy before pricing by simulation.
- Stochastic Programming: Barrera-Esteve, et al.[2006].
- Quantization: Bally et al. [2005], Bardou et al. [2007]. A quantization approach is implemented to price the Swing option without penalty.
- Pentanomial Tree: Wahab and Lee [2009]. A pentanomial tree approach is implemented to price swing options under GBM.

2 Issues Addressed in This Presentation

- Regime Switching Dynamics for the forward prices.
- Pentanomial tree approach to approximate the regime switching dynamics.
- Formulation of optimisation problem to account for make-up and carry-forward features under regime switching.
- Numerical implementations.

3 Basic Swing Contracts

- A basic swing contract is a contract for the supply of *daily* quantities of gas (within certain constraints) over a specified number of years at a specified set of contract prices. There is usually an annual contract quantity (ACQ_{T_i})
- Each *gas year* there is a minimum volume of gas (Take-or-Pay or Minimum Bill) which will be charged for regardless of the actual quantity of gas taken (MB_{T_i}).
- Each *day* of the gas year there is a maximum volume of gas which can be taken. Hence each *gas year* there is a maximum volume of gas which can be taken (MAX_{T_i}).

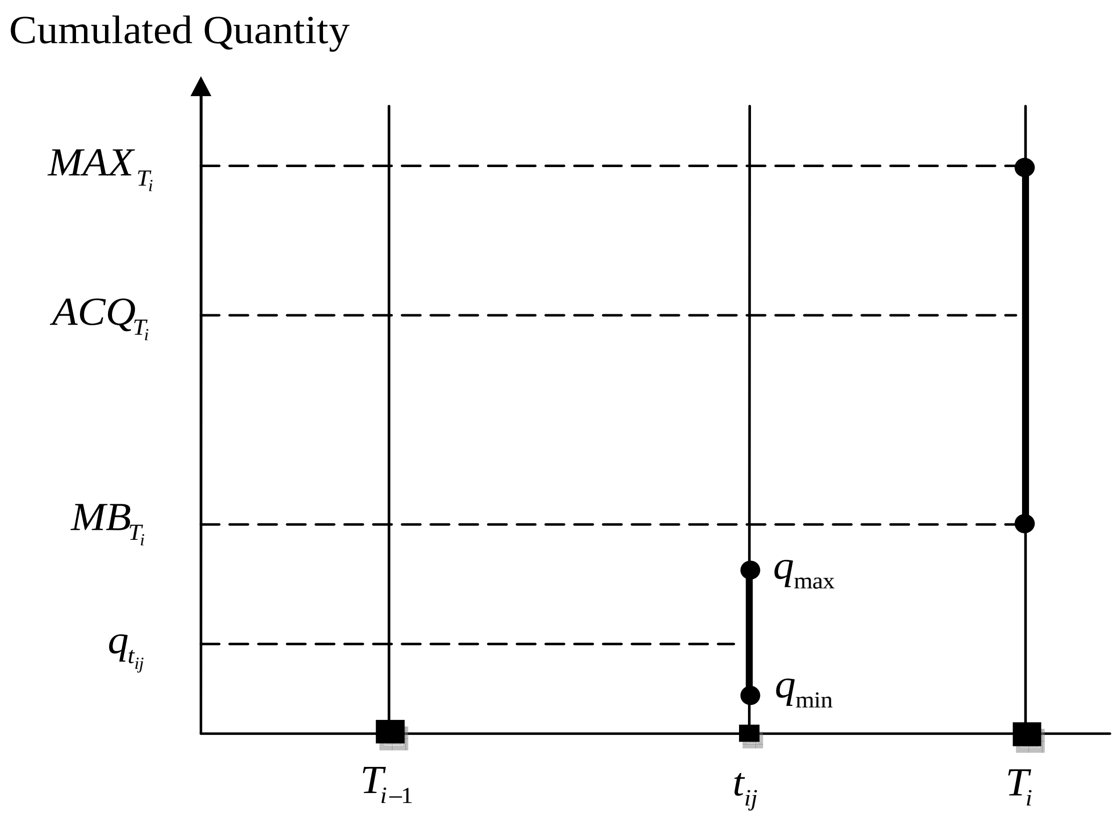

Figure 1: The Basic Swing Contract. T_i is year i, t_{ij} is jth day of year i

$$MB_{T_i} = \alpha_i . ACQ_{T_i}, \quad MAX_{T_i} \geq ACQ_{T_i} = \sum_{j=0}^{J} q_{t_{i,j}}$$

3.1 A Basic Take-or-Pay Contract as a Strip of Call Options

- A Take-or-Pay contract can be viewed as a variable volume swap or a strip of variable volume options with constraints.
- In the absence of a Take-or-Pay constraint

$$\textbf{Minimum Bill} = \mathbf{0}$$

  the optimal strategy each day is to purchase the max. allowable quantity when the market price is above the contract purchase price and nothing otherwise.

- In this case the contract has the maximum amount of flexibility and the value is equivalent to a strip of European Call options.

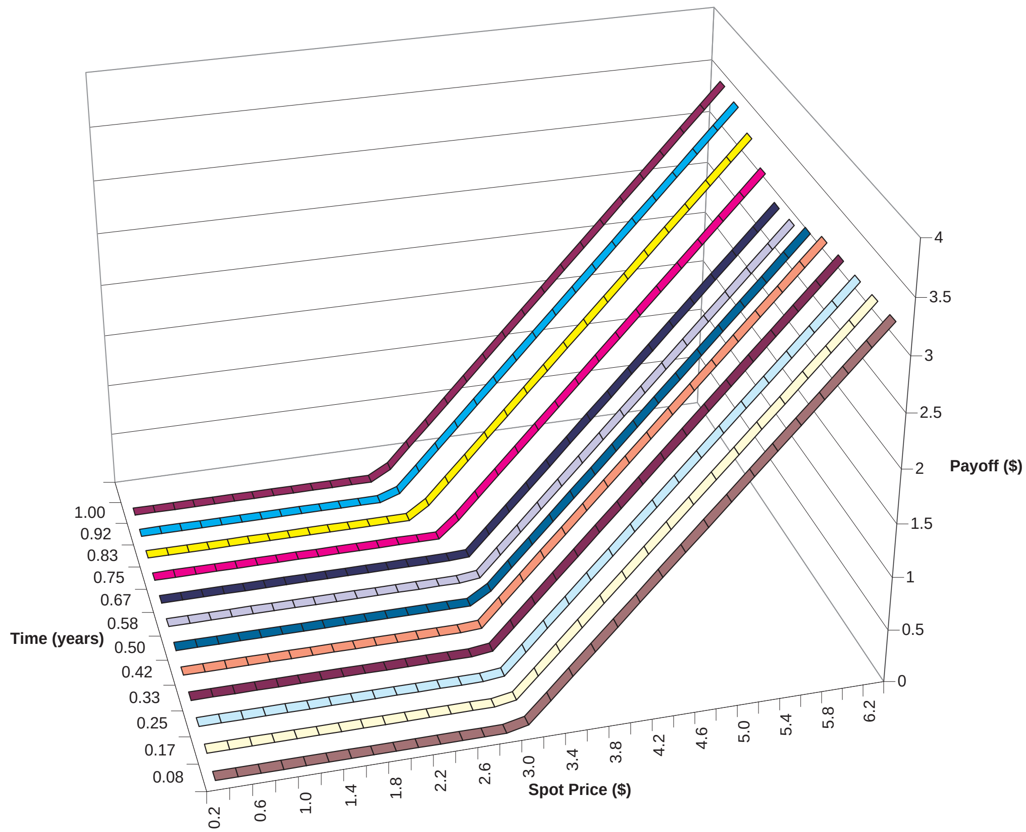

Figure 2: Payoff Diagram: Take-or-Pay as Strip of Call Options.

3.2 A Basic Take-or-Pay Contract as a Swap

- If

Minimum Bill = Maximum Annual Quantity,

the optimal daily strategy typically is to purchase the maximum allowable quantity regardless of the market price (depends on form of penalty).

- Now the contract is equivalent to a swap and has the minimum amount of flexibility and value.

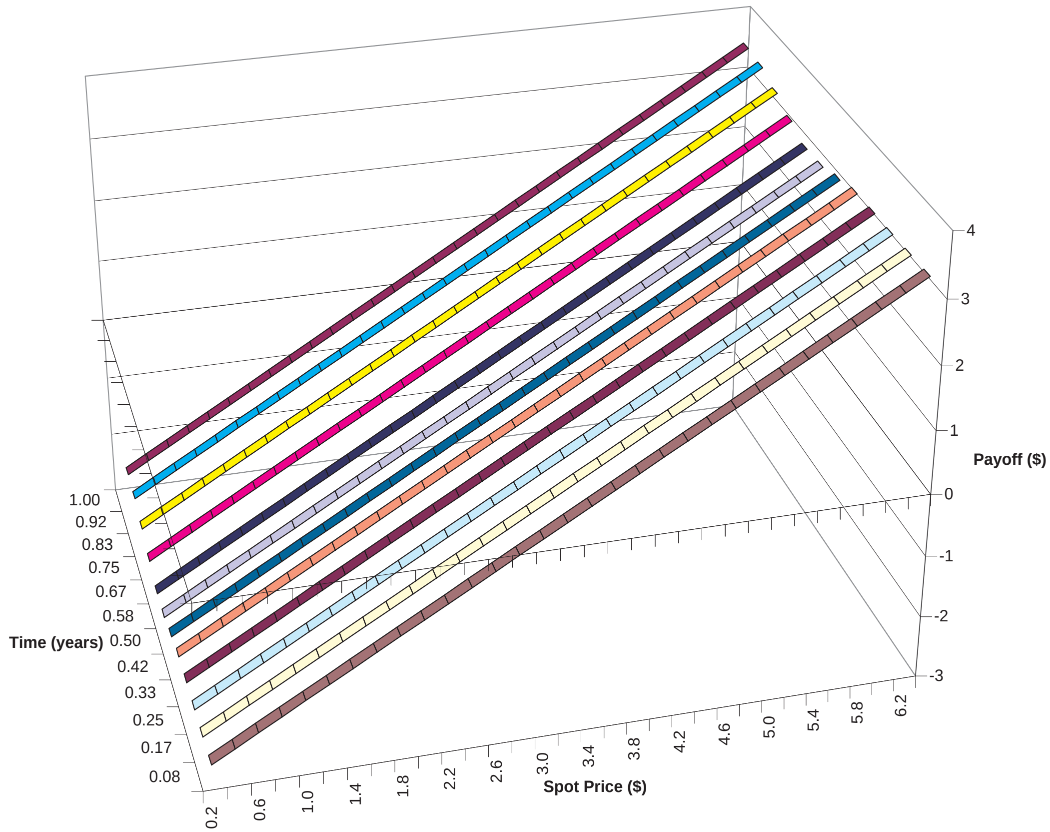

Figure 3: Payoff Diagram: Take-or-Pay as Swap

3.3 A Take-or-Pay Contract is a Combination of Call Strip and a Swap

- If

$$\mathbf{0 < \text{Minimum Bill} < \text{Maximum Annual Quantity,}}$$

  then the optimal strategy is to exercise like a strip of call options until the time left (to end of contract) is just sufficient to reach min. bill by taking the max. each day.

- In the constrained region there is a critical spot price (maybe less than the contract price) above which it is optimal to take the max. daily quantity, even though this results in a loss relative to the spot price case.

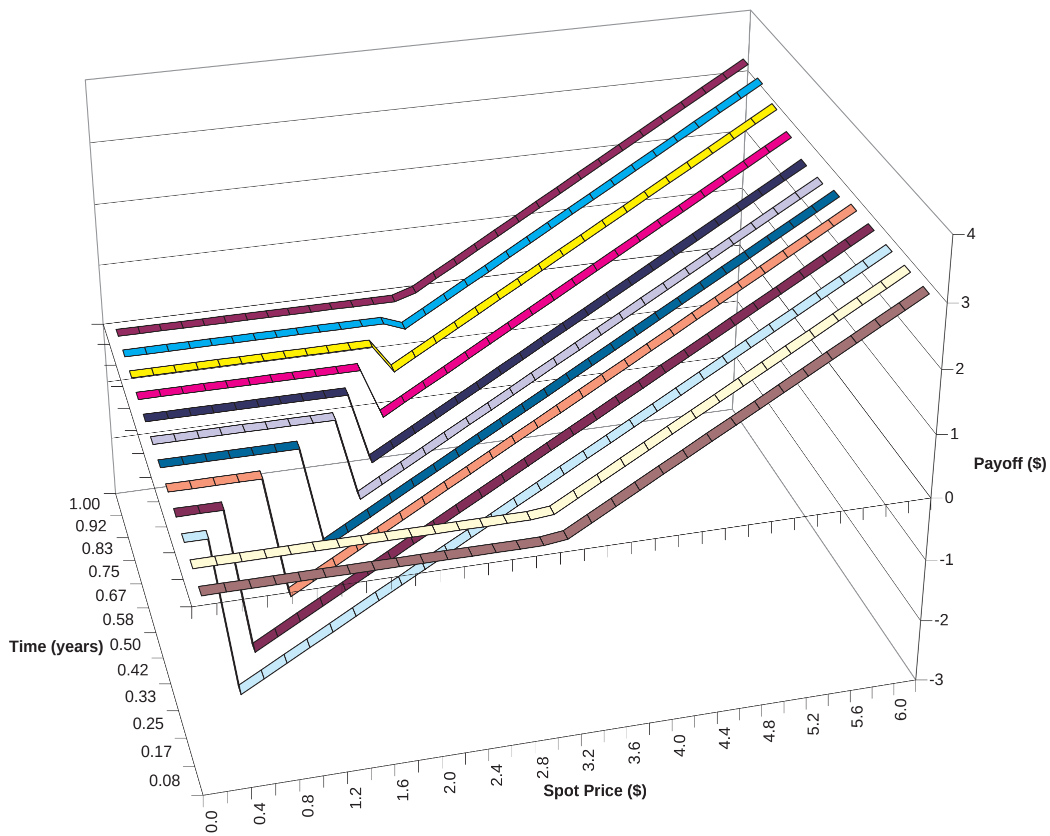

Figure 4: Payoff Diagram: Take-or-Pay as Combination of a Swap and Call Strip.

3.4 Swing Contracts with Make-Up and Carry Forward

- **Make-Up**
  - In years where the gas taken is less than Minimum Bill the shortfall (paid for in current year) is added to the *Make-Up Bank* (M_{T_i}).
  - In later years where the gas taken is greater than some reference level (typically Minimum Bill or ACQ) additional gas can be taken from the Make-Up Bank and a refund paid.
- **Carry Forward**
  - In years where the gas taken is greater than some reference level (typically ACQ) the excess gas is added to the *Carry Forward Bank* (C_{T_i}).
  - In later years Carry Forward Bank gas can be used to reduce the Minimum Bill for that year.

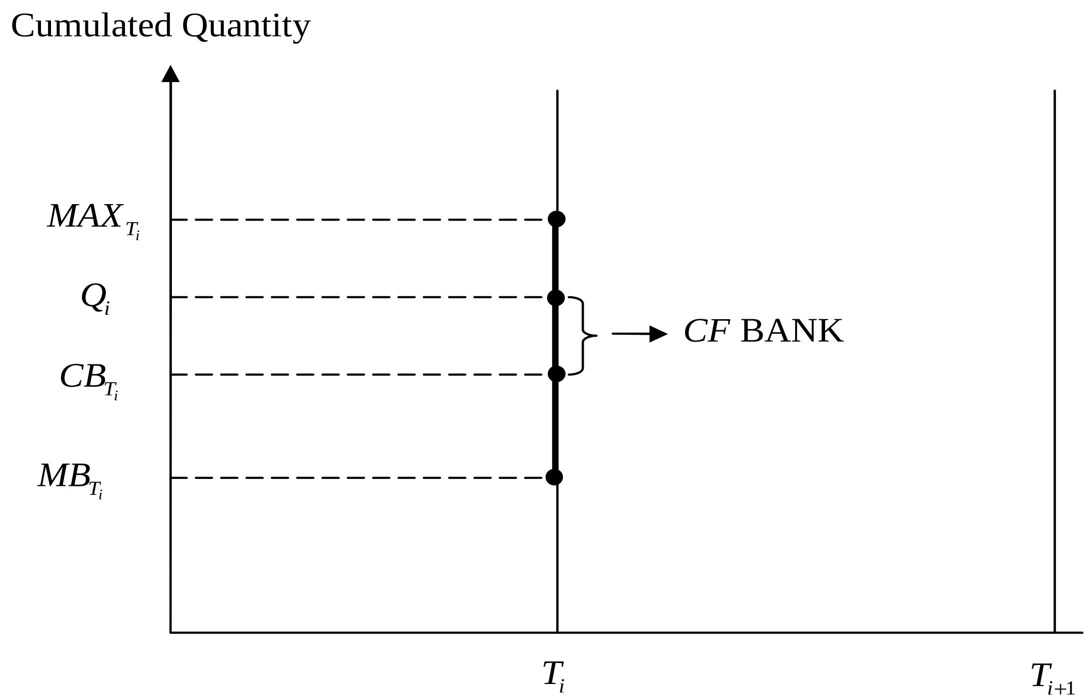

Figure 5: Carry Forward Bank

$$Q_i = \text{quantity taken in year } i$$

$$CB_{T_i} = \text{carry forward base in year } i$$

$$C_{T_i} = (1 - \beta_{i-1})C_{T_{i-1}} + \max\{Q_i - CB_{T_i}, 0\}$$

[**evolution of carry forward bank**]

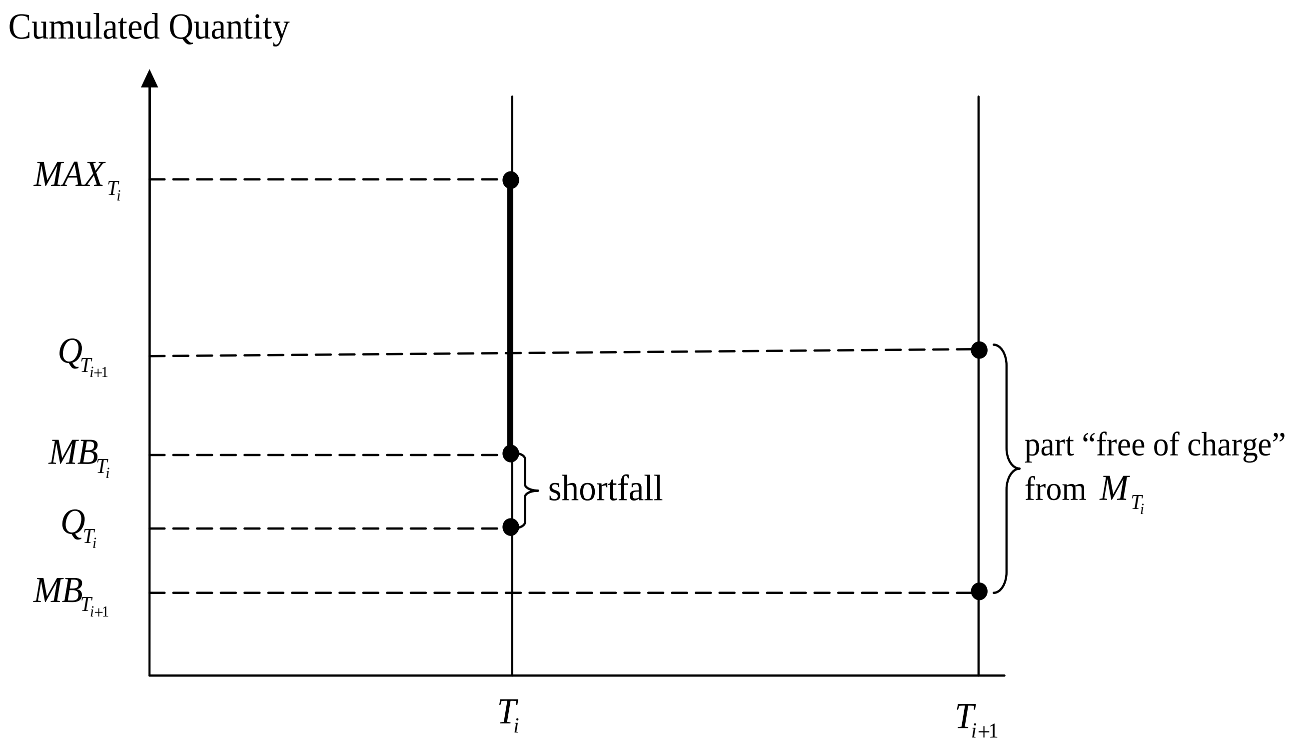

Figure 6: Make Up Bank; (β_i, γ_i) control variables

$$MB_{T_i} = MB_{T_i}^{(0)} - \beta_i C_{T_i}$$ [**use carry-forward bank to reduce min. bill**]

$$M_{T_i} = (1 - \gamma_{i-1}) M_{T_{i-1}} + \max(MB_{T_i} - Q_{T_i}, 0)$$ [**evolution of make-up bank**]

4 Forward Price Curve with Regime Switching

- The stochastic or random nature of commodity prices plays a central role in the models for valuing financial contingent claims, for example, swing options on commodities and gas storage contracts.
- The observed quantity – $F(t, T)$

$$F(t, T) = \text{forward price at time } t \text{ for delivery of gas at time } T.$$

- Those contracts are widely traded on many exchanges with prices readily observed.
- The nearest maturity forward price is used as a proxy for the spot price.
- The longer dated contracts are used to imply the convenience yield.

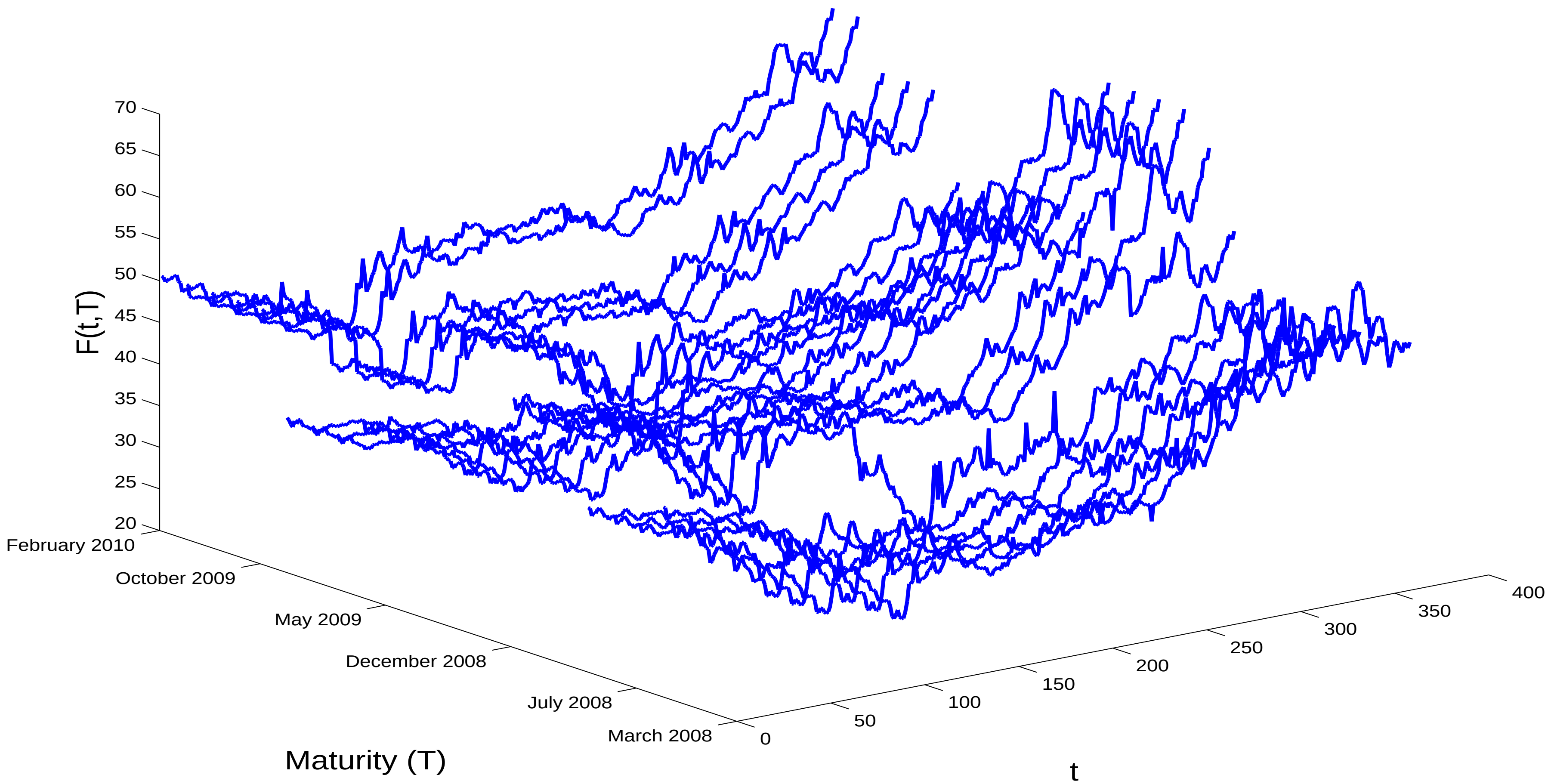

Figure 7: Forward price curves of 24 different maturities from March 2008 to February 2010 with data from 29/09/2006 to 19/02/2008.

4.1 Stochastic volatility needed

- Deterministic volatility models — the volatility curve is fixed and the volatility of a specific forward price can change deterministically only with maturity.
- To properly describe the actual evolution of the volatility curve, one needs a process consisting of both deterministic and random factors.
- The drawback of diffusion models is that they **cannot generate sudden and sufficiently large shifts of the volatility curve**.
- Adding traditional type jump processes, for example Poisson jumps, one finds that, **the frequency of the jumps is too large while the magnitude of the jumps is too small**.

4.2 Regime Switching is better

- An appropriate framework for modelling the dynamics of volatilities: a class of **piecewise-deterministic processes** which allow volatility to follow an almost deterministic process between two random jump times.
- The simplest process in this class is the continuous-time homogeneous Markov chain with a finite number of jump times. Models with such a process approximate the actual jumps in volatility with jumps over a finite set of values.
- **Hidden Markov Model (HMM) - EM Algorithm**, **Markov Chain Monte Carlo (MCMC)** approach are able to estimate the parameters of such models.

4.3 Regime Switching Forward Price Curve

We use the following model for the forward prices in the natural gas market.

$$
\begin{aligned}
\frac{dF(t,T)}{F(t,T)} &= \sigma_1(t,T)dW_1(t) + \sigma_2(t,T)dW_2(t), \\
\sigma_1(t,T) &= <\sigma_1, X_t> c(t)\left(e^{-<\alpha_1, X_t>(T-t)}(1-\sigma_{l_1}) + \sigma_{l_1}\right), \\
\sigma_2(t,T) &= <\sigma_2, X_t> c(t)\left(\sigma_{l_2} - e^{-<\alpha_2, X_t>(T-t)}\right), \\
c(t) &= c + \sum_{j=1}^{J} \left(d_j(1+\sin(f_j + 2\pi j t))\right).
\end{aligned}
$$

4.4 One Factor Model: An Example

- To price the gas swing contract, we consider the one factor model:

$$\frac{dF(t,T)}{F(t,T)} = < \sigma, X_t > c(t) \cdot e^{-\alpha(T-t)} dW_t,$$

where W_t is a standard BM and X_t is a finite state Markov Chain and $c(t) = c + \sum_{j=1}^{J} (d_j(1 + \sin(f_j + 2\pi j t)))$ captures the seasonal effect.

- Here, the spot volatility σ will take different values depending on the state of the Markov Chain X_t. Consequently, the spot price will follow:

$$S(t) = F(0,t) \cdot \exp\left(\int_0^t < \sigma, X_s > c(s) \cdot e^{-\alpha(t-s)} dW_s - \frac{1}{2}\Lambda_t^2 \right),$$

where $\Lambda_t^2 = \int_0^t (< \sigma, X_s > c(s) \cdot e^{-\alpha(t-s)})^2 ds.$

5 Pentanomial Tree Construction

- Bollen (1998) constructed a pentanomial lattice to approximate a regime switching GBM and to price both European and American options.
- Wahab and Lee (2009) extended the pentanomial lattice to a multinomial tree and studied the price of swing options under the regime switching GBM dynamics.
- To construct a discrete pentanomial lattice approximating the spot price process $S(t)$, we let $Y_t = \int_0^t < \sigma, X_s > c(s) \cdot e^{-\alpha(t-s)} dW_s$.
- We build a discrete lattice to approximate Y_t first, we know that:

$$dY_t = -\alpha Y_t dt + < \sigma, X_t > c(t) dW_t.$$

5.1 Nodes

- Assume there are only two regimes for the volatility, namely low volatility σ_L and high volatility σ_H.
- In pentanomial tree in Figure 9, each regime is represented by a trinomial tree with one branch being shared by both regimes.
- In order to minimize the number of nodes in the tree, nodes from both regimes are merged by setting the step sizes of both regimes at a $1 : 2$ ratio.

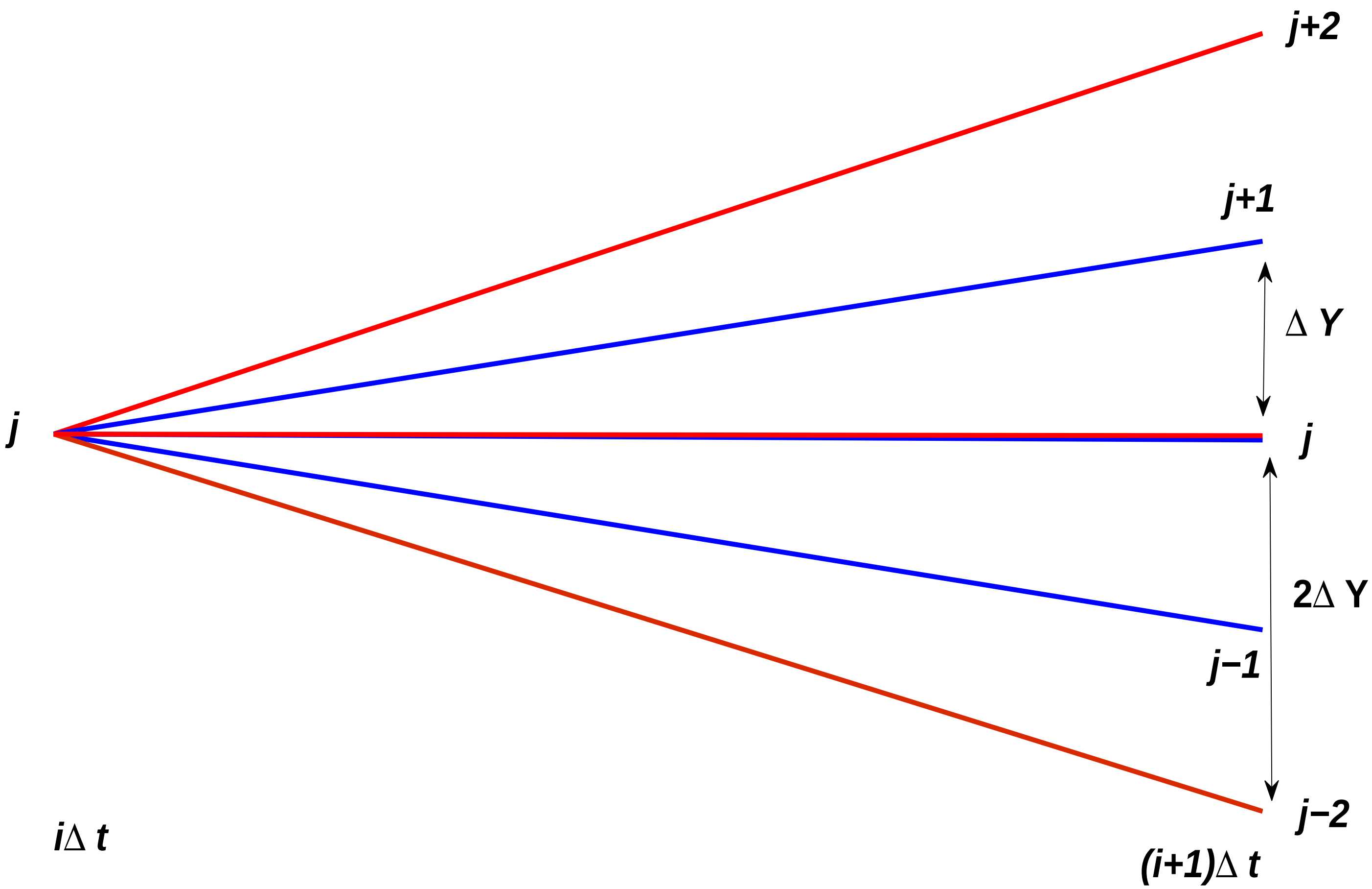

Figure 8: The recombining of a pentanomial tree.

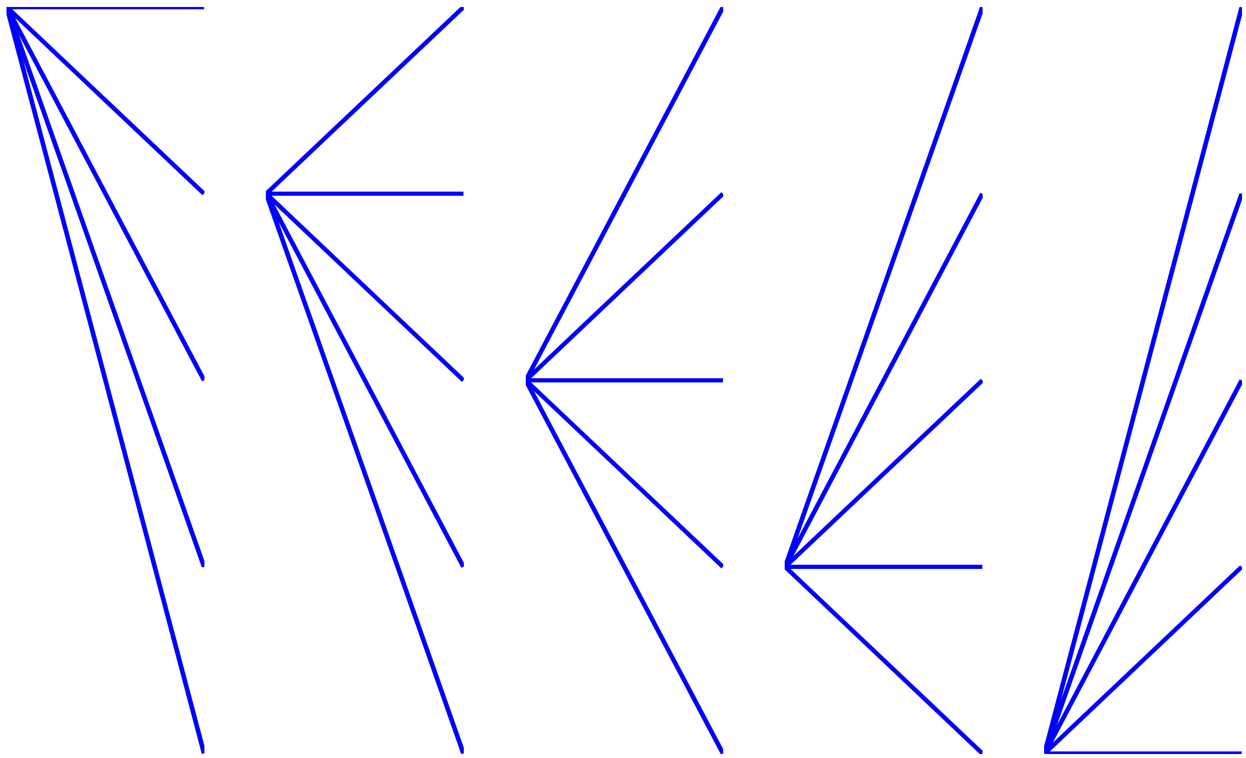

Figure 9: The Alternative Branching Processes for the mean reverting processes. The level where the tree switches from one branching to another depends on the attenuation parameter α and the time step Δt.

- The time values in the tree is $t_i = i\Delta t$, where Δt is the time step.
- The levels of Y are equally spaced and have the form $Y_{i,j} = j\Delta Y$, where ΔY is the space step.
- Any node in the tree can therefore be referenced by a pair of integers (i, j) that is the node at the $i-$th time step and $j-$th level.
- From stability and convergence considerations, a reasonable choice for the relationship between the space step ΔY and the time step Δt is given by (see Wahab and Lee (2009)):

$$\Delta Y = \begin{cases} \sigma_L \sqrt{3\Delta t}, & \sigma_L \geq \sigma_H/2; \\ \frac{\sigma_H}{2}\sqrt{3\Delta t}, & \sigma_L < \sigma_H/2. \end{cases}$$

5.2 Transition probabilities

- The trinomial branching process and the associated probabilities are chosen to be consistent with the conditional drift and variance of the process.
- When the volatility is in the low regime, $\sigma = \sigma_L$, looking at the inner trinomial tree, we want to match:

$$E[\Delta Y] = -\alpha Y_{i,j}\Delta t,\ E[\Delta Y^2] = \sigma_L^2 \Delta t + E[\Delta Y]^2;$$

equating the first and second moments of ΔY in the tree we have:

$$p^L_{u,i,j}((k+1)-j) + p^L_{m,i,j}(k-j) + p^L_{d,i,j}((k-1)-j) = -\alpha Y_{i,j}\Delta t/\Delta Y,$$

$$p^L_{u,i,j}((k+1)-j)^2 + p^L_{m,i,j}(k-j)^2 + p^L_{d,i,j}((k-1)-j)^2 = (\sigma_L^2 \Delta t + (-\alpha Y_{i,j}\Delta t)^2)/\Delta Y^2,$$

together with $p^L_{u,i,j} + p^L_{m,i,j} + p^L_{d,i,j} = 1$ we can obtain

$$p^L_{u,i,j} = \frac{1}{2}\left[\frac{\sigma_L^2\Delta t + \alpha^2 Y_{i,j}^2 \Delta t^2}{\Delta Y^2} + (k-j)^2 - \frac{\alpha Y_{i,j}\Delta t}{\Delta Y}(1 - 2(k-j)) - (k-j)\right],$$

$$p^L_{d,i,j} = \frac{1}{2}\left[\frac{\sigma_L^2\Delta t + \alpha^2 Y_{i,j}^2 \Delta t^2}{\Delta Y^2} + (k-j)^2 + \frac{\alpha Y_{i,j}\Delta t}{\Delta Y}(1 + 2(k-j)) + (k-j)\right],$$

$$p^L_{m,i,j} = 1 - p^L_{u,i,j} - p^L_{d,i,j}.$$

- When the volatility is in high regime, $\sigma = \sigma_H$,we will have:

$$p^H_{u,i,j} = \frac{1}{8}\left[\frac{\sigma_H^2 \Delta t + \alpha^2 Y_{i,j}^2 \Delta t^2}{\Delta Y^2} + (k-j)^2 - \frac{\alpha Y_{i,j} \Delta t}{\Delta Y}(2 - 2(k-j)) - 2(k-j)\right],$$

$$p^H_{d,i,j} = \frac{1}{8}\left[\frac{\sigma_H^2 \Delta t + \alpha^2 Y_{i,j}^2 \Delta t^2}{\Delta Y^2} + (k-j)^2 + \frac{\alpha Y_{i,j} \Delta t}{\Delta Y}(2 + 2(k-j)) + 2(k-j)\right],$$

$$p^H_{m,i,j} = 1 - p^H_{u,i,j} - p^H_{d,i,j}.$$

5.3 State prices for both regimes

- We will displace the nodes in the above simplified tree by adding the proper drifts a_i which are consistent with the observed forward prices.
- For $x = L, H$ we define state prices $Q^x_{i,j}$ as the present value of a security that pay off $\$1$ if $Y = j\Delta Y$ and $X_{i\Delta t} = x$ at time $i\Delta t$ and zero otherwise.
- Hence those state prices are accumulated according to

$$Q^L_{0,0} = 1,\ Q^H_{0,0} = 0;\ \text{for lower volatility regime}$$

$$Q^L_{0,0} = 0,\ Q^H_{0,0} = 1;\ \text{for higher volatility regime}$$

$$Q^L_{i+1,j} = \sum_{j'} (Q^L_{i,j'} p^X_{L,L} + Q^H_{i,j'} p^X_{H,L}) p^L_{j',j} P(i\Delta t, (i+1)\Delta t);$$

$$Q^H_{i+1,j} = \sum_{j'} (Q^L_{i,j'} p^X_{L,H} + Q^H_{i,j'} p^X_{H,H}) p^H_{j',j} P(i\Delta t, (i+1)\Delta t);$$

- Where $p^X_{x,x'}$ is the probabilities the Markov Chain transits from the state x to the state x' and $p^L_{j',j}$ and $p^H_{j',j}$ are the probabilities the spot transits from j' to j but arriving at low and high volatility regime respectively and $P(i\Delta t, (i+1)\Delta t)$ denotes the price at time $i\Delta t$ of the pure discount bond maturing at time $(i+1)\Delta t$.

- To use the state prices to match the forward price curve we use:

$$P(0, i\Delta t) F(0, i\Delta t) = \sum_j (Q^L_{i,j} + Q^H_{i,j}) S_{i,j},$$

- Hence the adjustment needed to ensure the tree correctly returns the observed futures curve can be calculated.

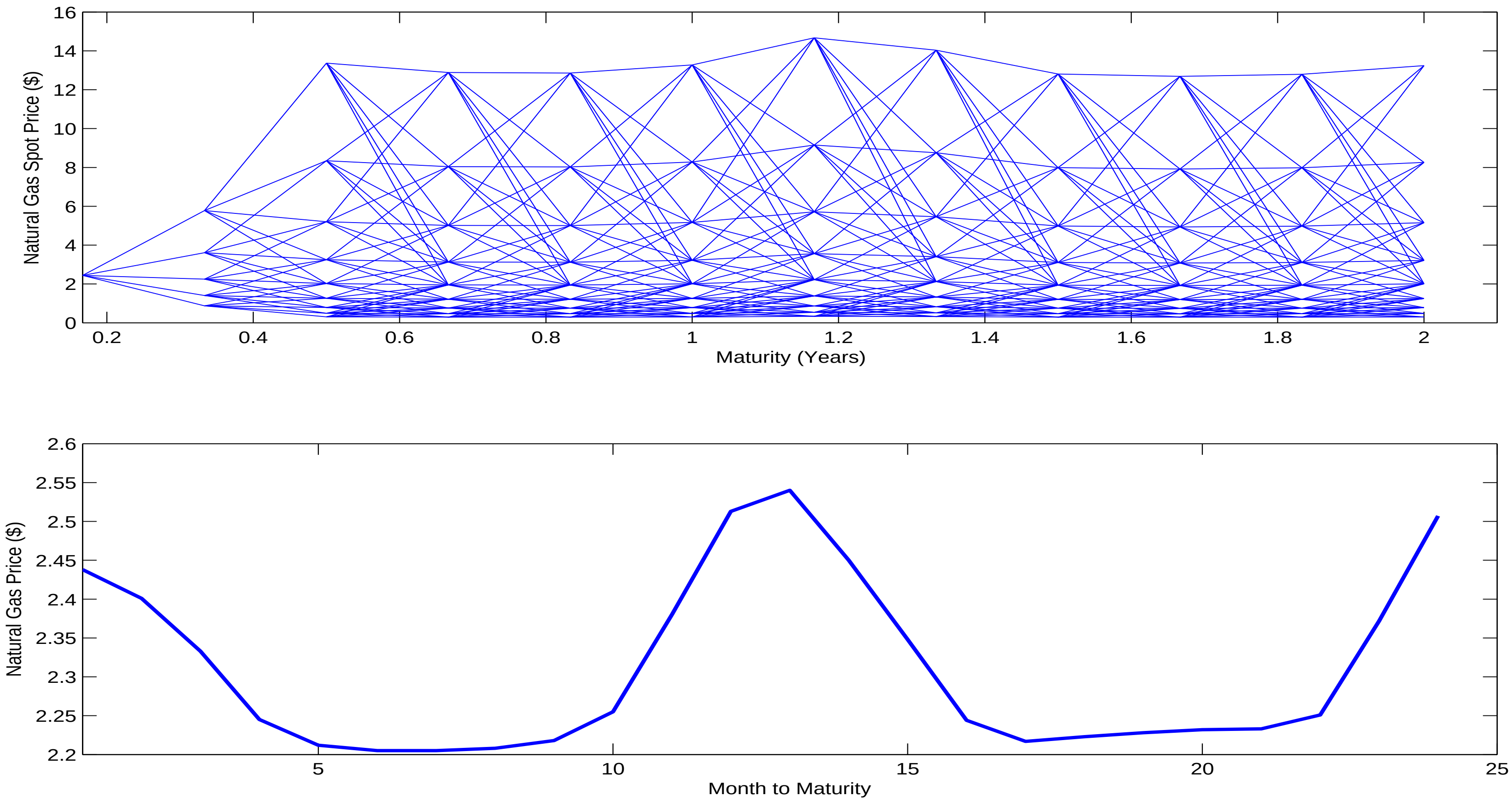

Figure 10: Spot Price Tree which is consistent with the Seasonal Forward Curve.

6 Evaluation of Swing Contract

- Let $V_t^*(S,Q,i)$ and $q_t^*(S,Q,i), t = 0,1,\ldots,T$ be the time t value and decision function of a Take-or-Pay contract when the spot price is S, the period-to-date consumption is Q and the system is in regime i.
- MB - Minimum Bill; K - Contract Price.
- Optimal decisions $(q_T^*(S,Q,i))$ and optimal value functions $(V_T^*(S,Q,i))$ at the maturity of the contract are as follows

$$q_T^*(S,Q,i) = \begin{cases} 1, & S > K; \\ \min(\max(MB - Q, 0), 1), & S \leq K. \end{cases}$$

$$V_T^*(S,Q,i) = (S-K)q_T^*(S,Q,i) - K\max(0, Q + q_T^*(S,Q,i) - MB).$$

- For $t = T-1, \cdots, 0,$ working backward in time we have:

$$V_t^*(S,Q,i) =$$
$$\max_{q\in[0,1]} \left\{ q(S-K) + e^{-rdt}\sum_{j=1}^{N} p_{ij} E_S^i[V_{t+1}^*(S_{t+1}, Q+q, j)] \right\};$$

$$q_t^*(S,Q,i) =$$
$$\text{argmax}_q \left\{ q(S-K) + e^{-rdt}\sum_{j=1}^{N} p_{ij} E_S^i[V_{t+1}^*(S_{t+1}, Q+q, j)] \right\}.$$

together with the following boundary conditions:

$$V_t^*(S, Q_{max}, i) = 0, \;\; q_t^*(S, Q_{max}, i) = 0,$$

which means that the value function will be zero and there is no gas to use if the period to date consumption reaches the maximal quantity.

7 Numerical Examples

One year take-or-pay contract price differences when

- Volatilities: $\sigma_L = 0.5, \sigma_H = 1.0$;
- Mean reversion rate: $\alpha = 5$;
- Forward curve: $F(0, t) = 100$;
- Interest rate: $r = 0$;
- Contract price: $K = 100$;
- Maturity time: $T = 365$.
- Minimal Bill: $MB = 365 \times 80\% = 292$;
- Transition matrix of the hidden MC: $P = \begin{bmatrix} 0.8516 & 0.1484 \\ 0.7080 & 0.2920 \end{bmatrix}$.

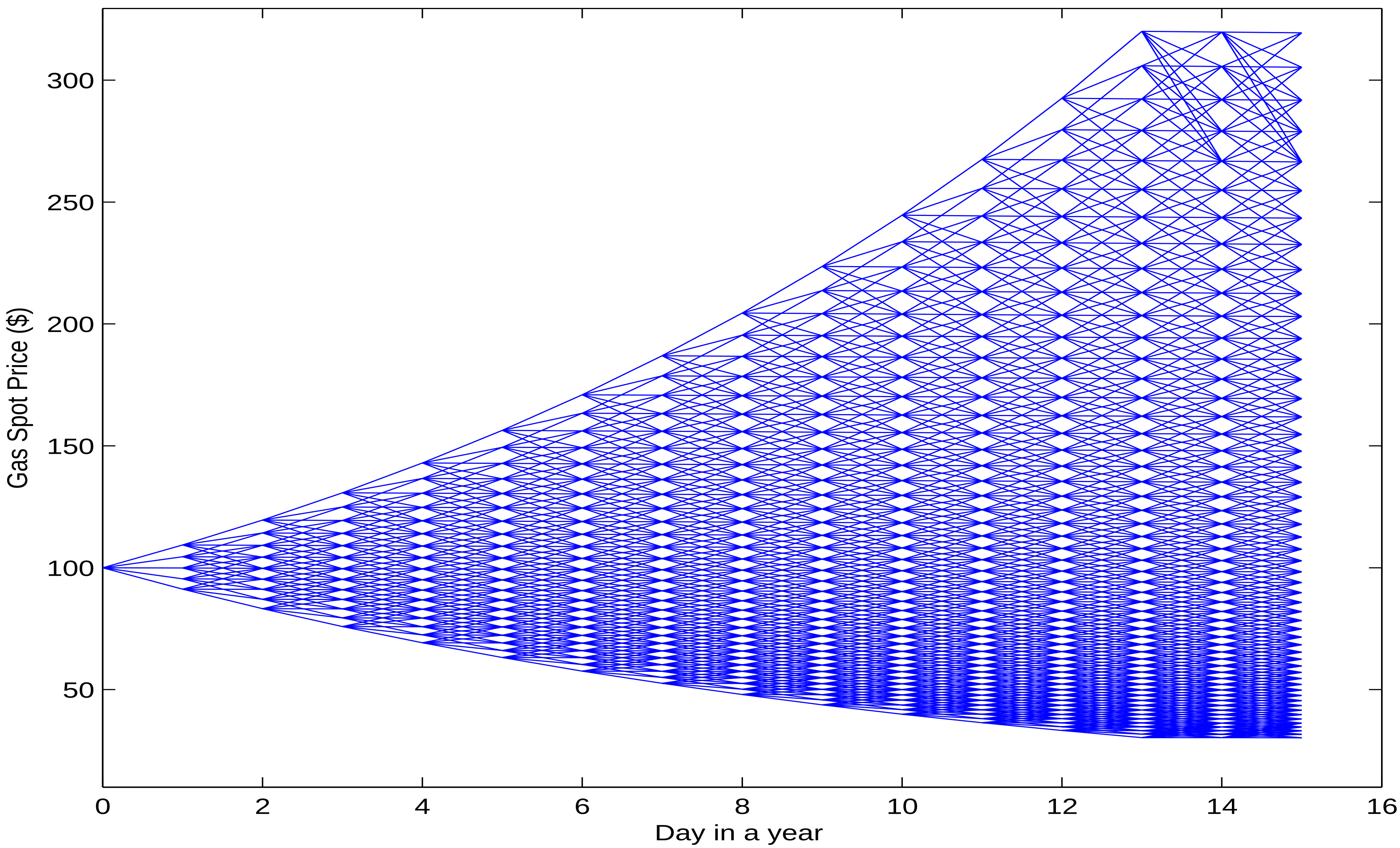

Figure 11: Part of the Pentanomail tree based on the above parameters.

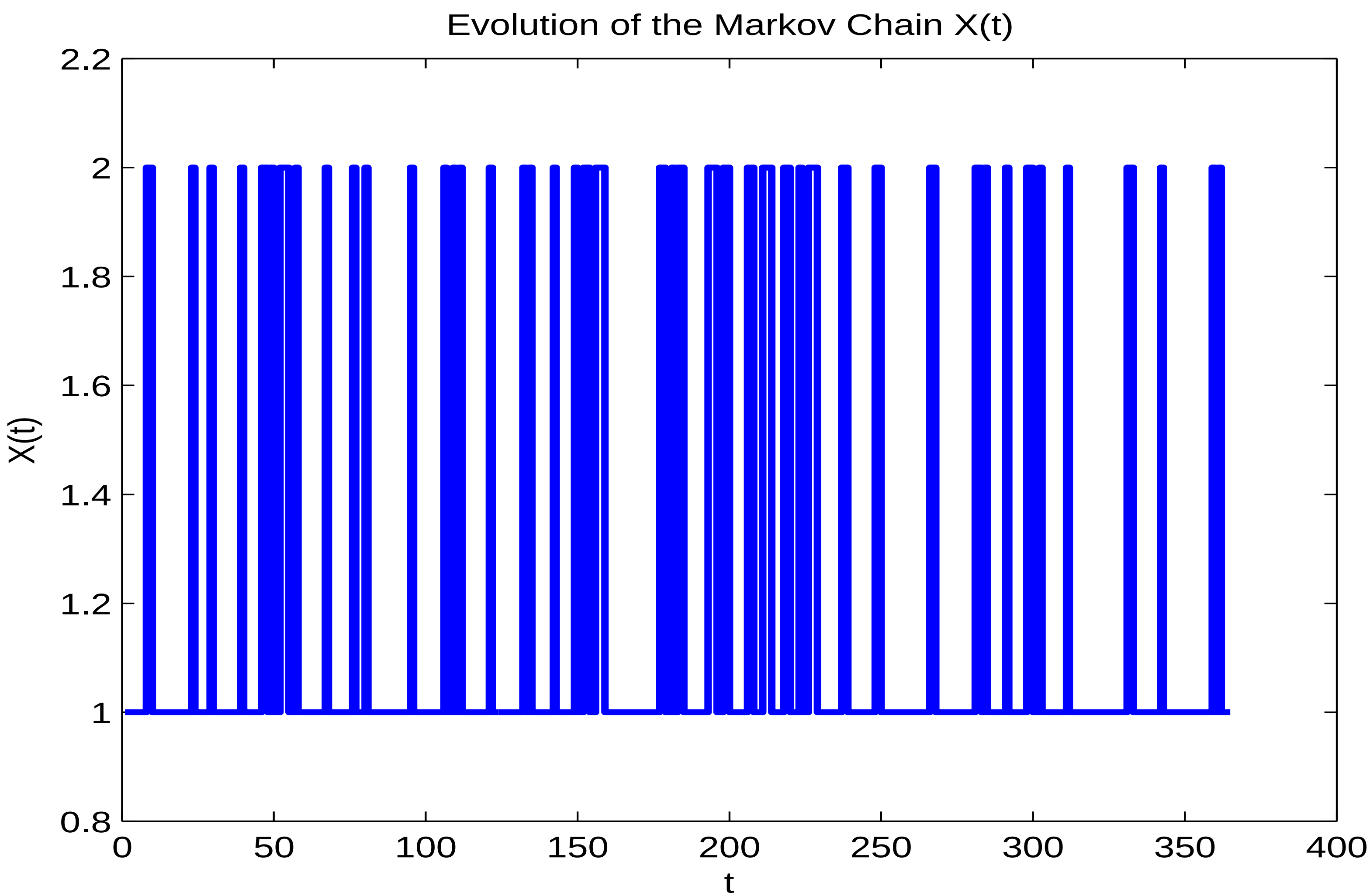

Figure 12: A typical evolution of Markov Chain $\boldsymbol{X}(\boldsymbol{t})$.

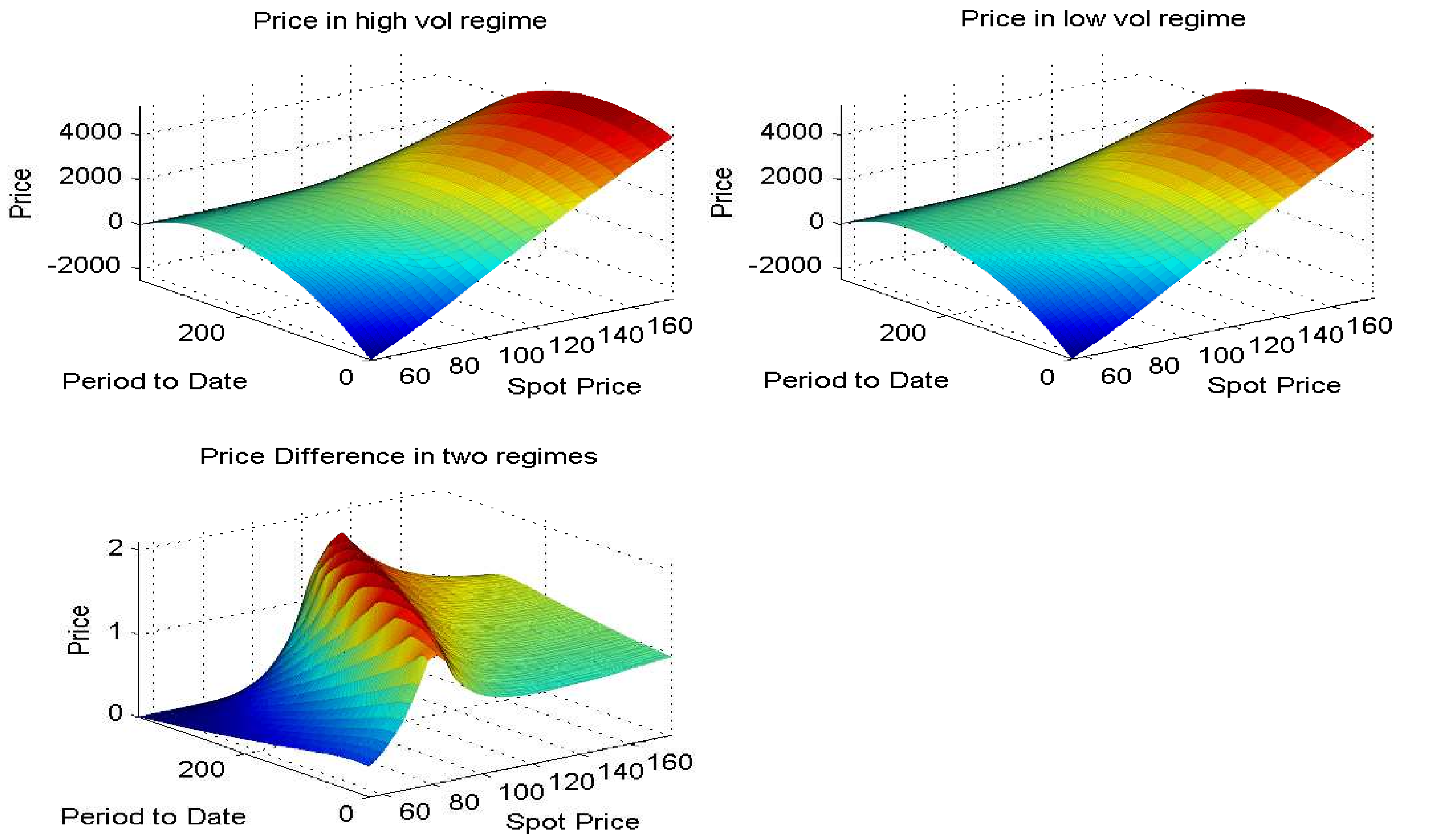

Figure 13: Day 0 price differences in two different regimes.

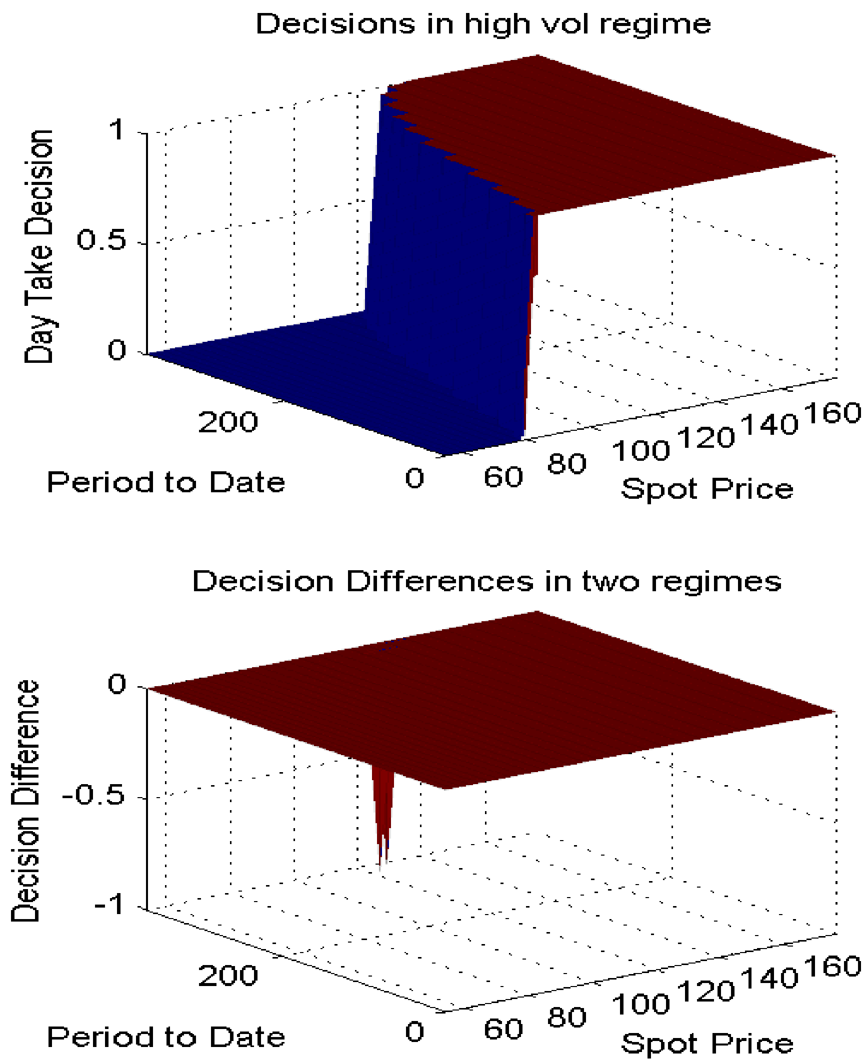

Figure 14: Day 0 decision differences in two different regimes.

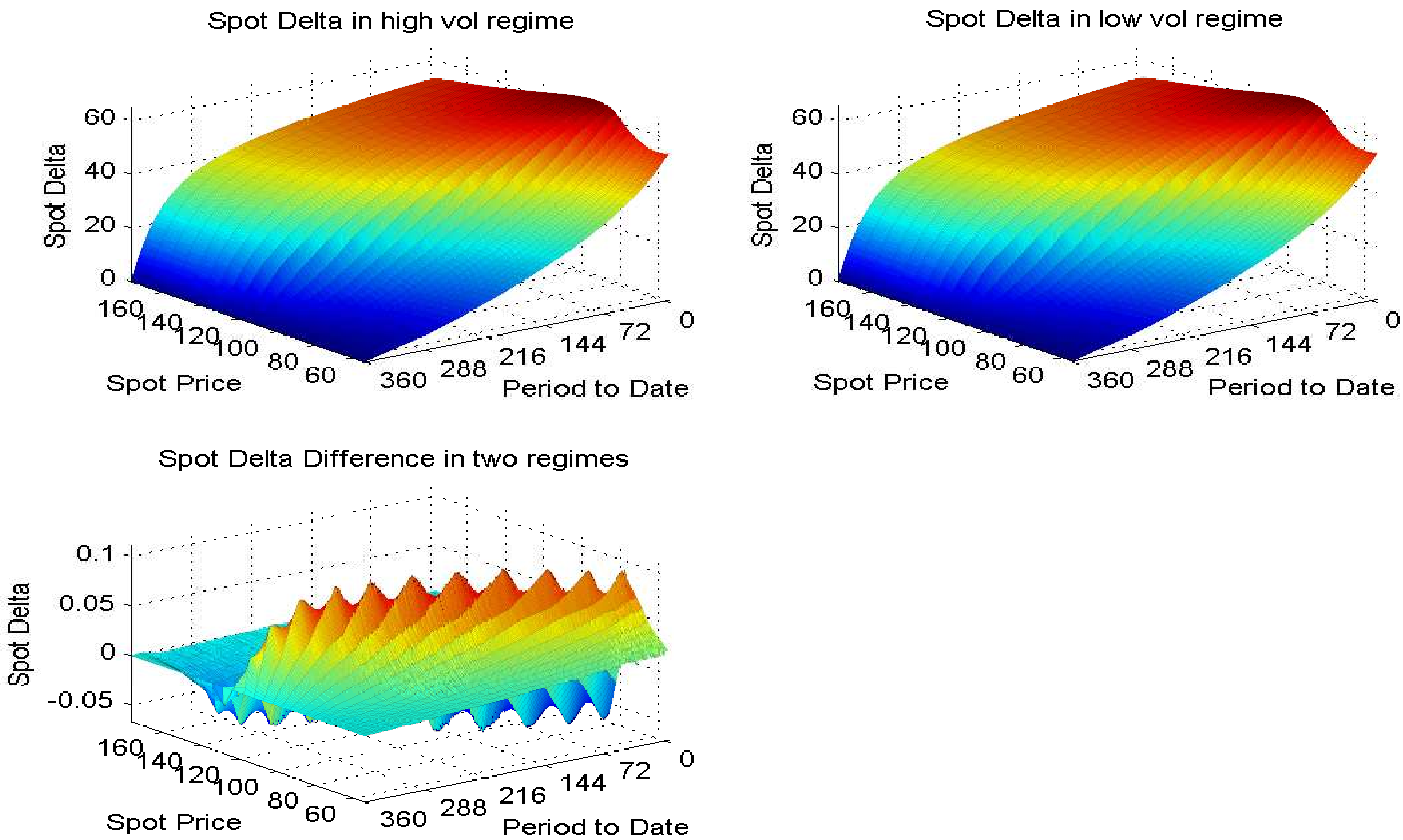

Figure 15: Day 0 spot delta differences in two different regimes.

One year take-or-pay contract price differences when

- Volatilities: $\sigma_L = 0.5, \sigma_H = 1.0$;
- Mean reversion rate: $\alpha = 5$;
- Forward curve: $F(0, t) = 100$;
- Interest rate: $r = 0$;
- Contract price: $K = 100$;
- Maturity time: $T = 365$;
- Minimal Bill: $MB = 365 \times 80\% = 292$;
- Transition matrix of the hidden MC: $P = \begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix}$.

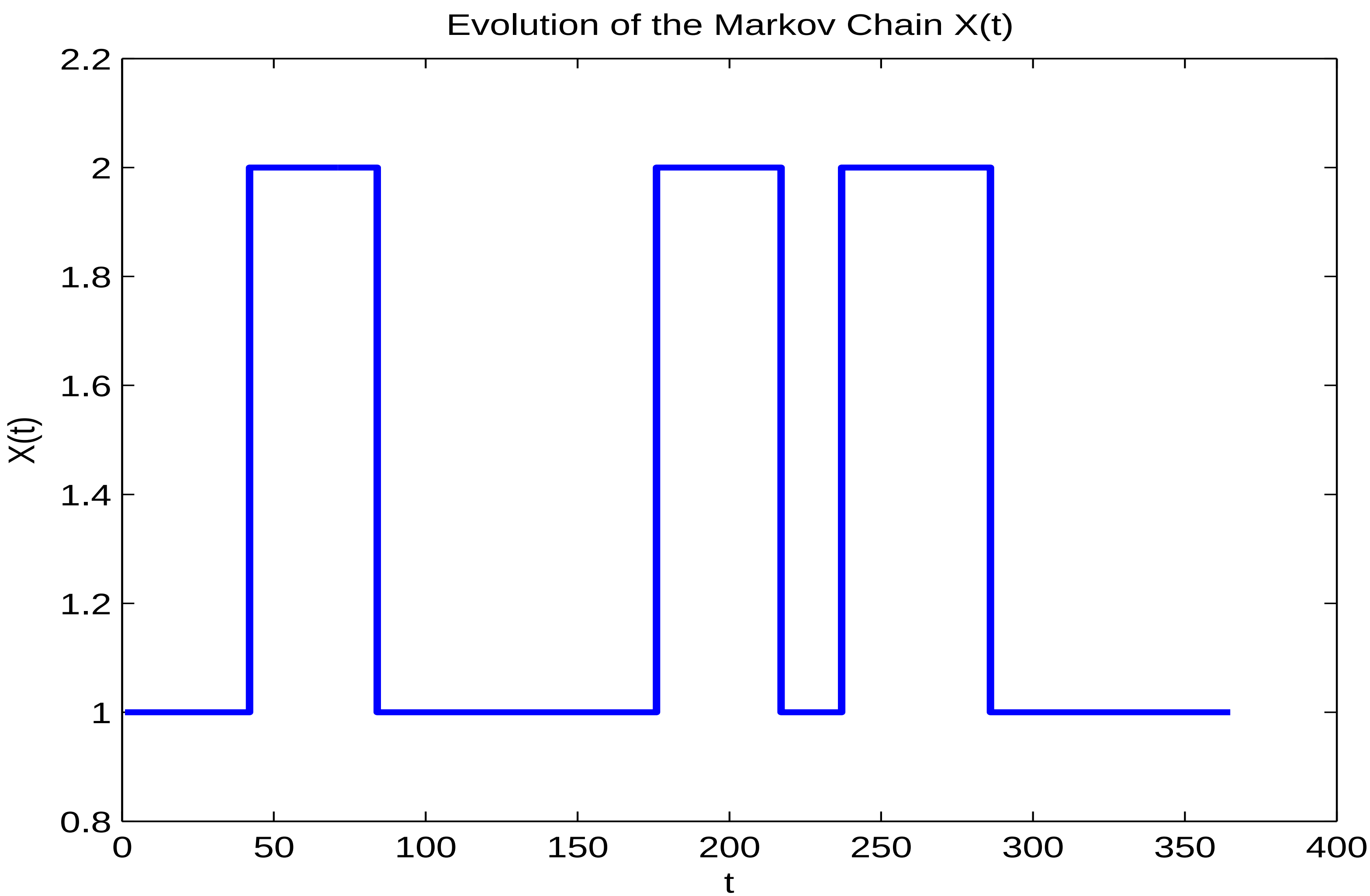

Figure 16: A typical evolution of Markov Chain $X(t)$.

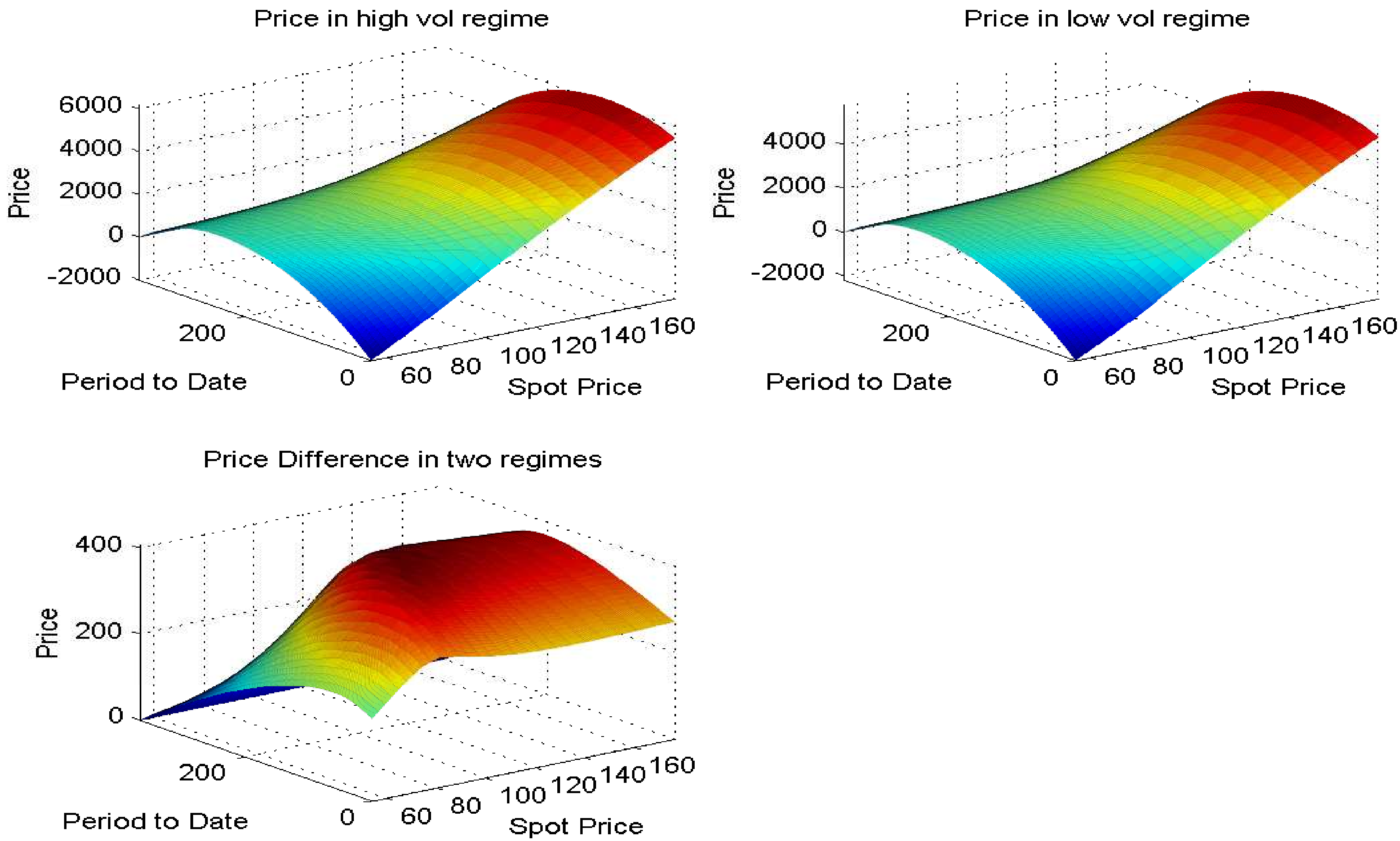

Figure 17: Day 0 price differences in two different regimes.

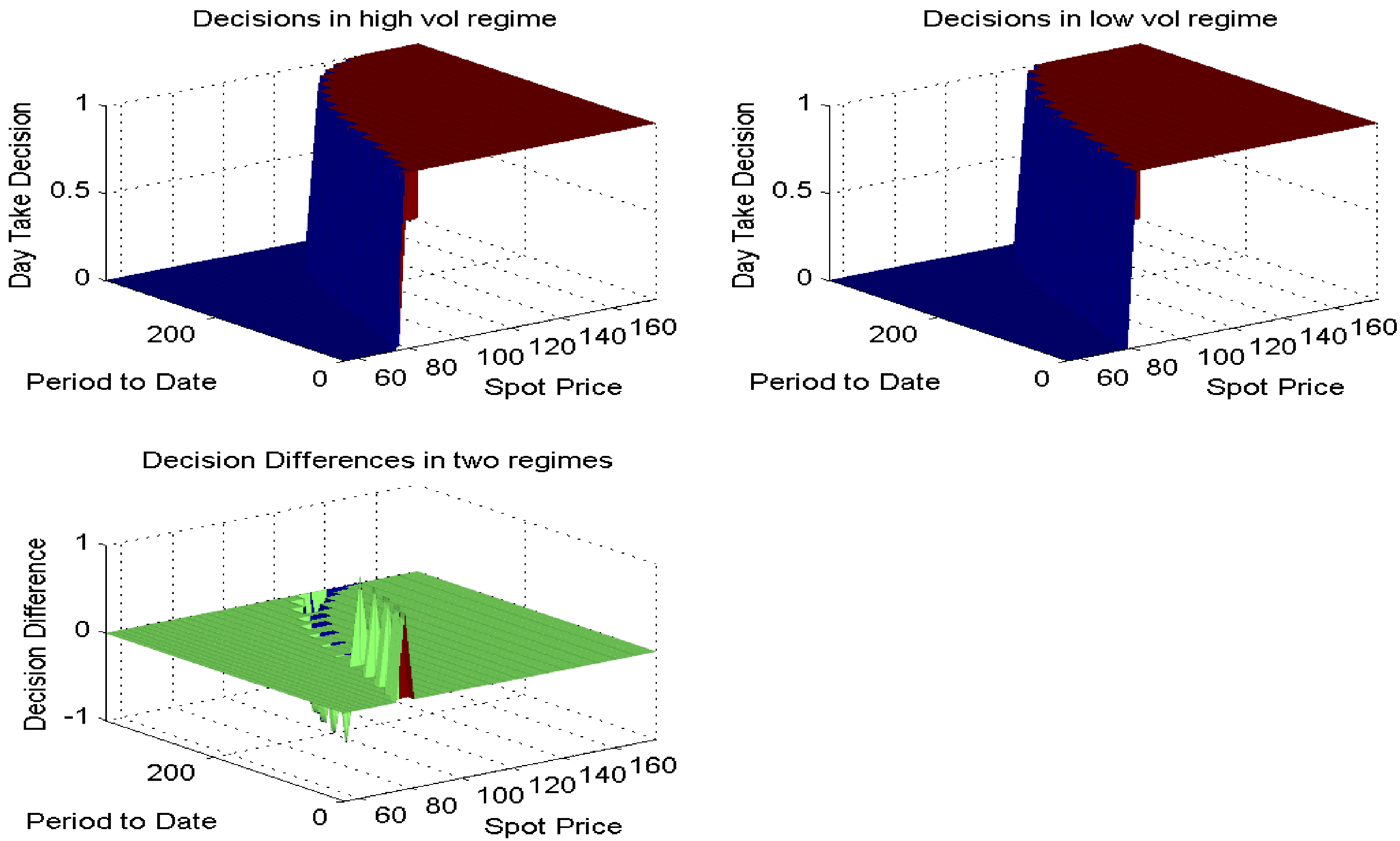

Figure 18: Day 0 decision differences in two different regimes.

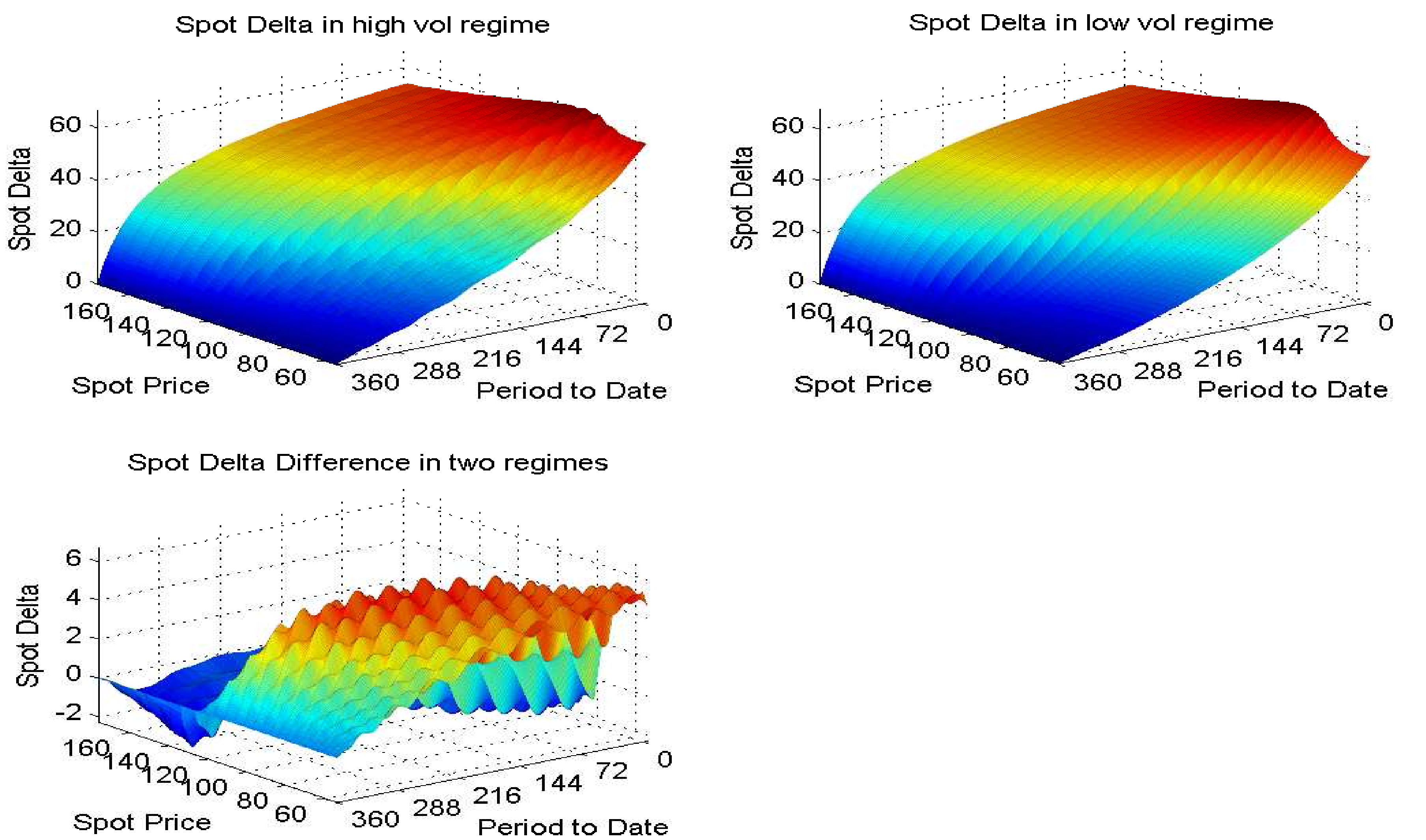

Figure 19: Day 0 spot delta differences in two different regimes.

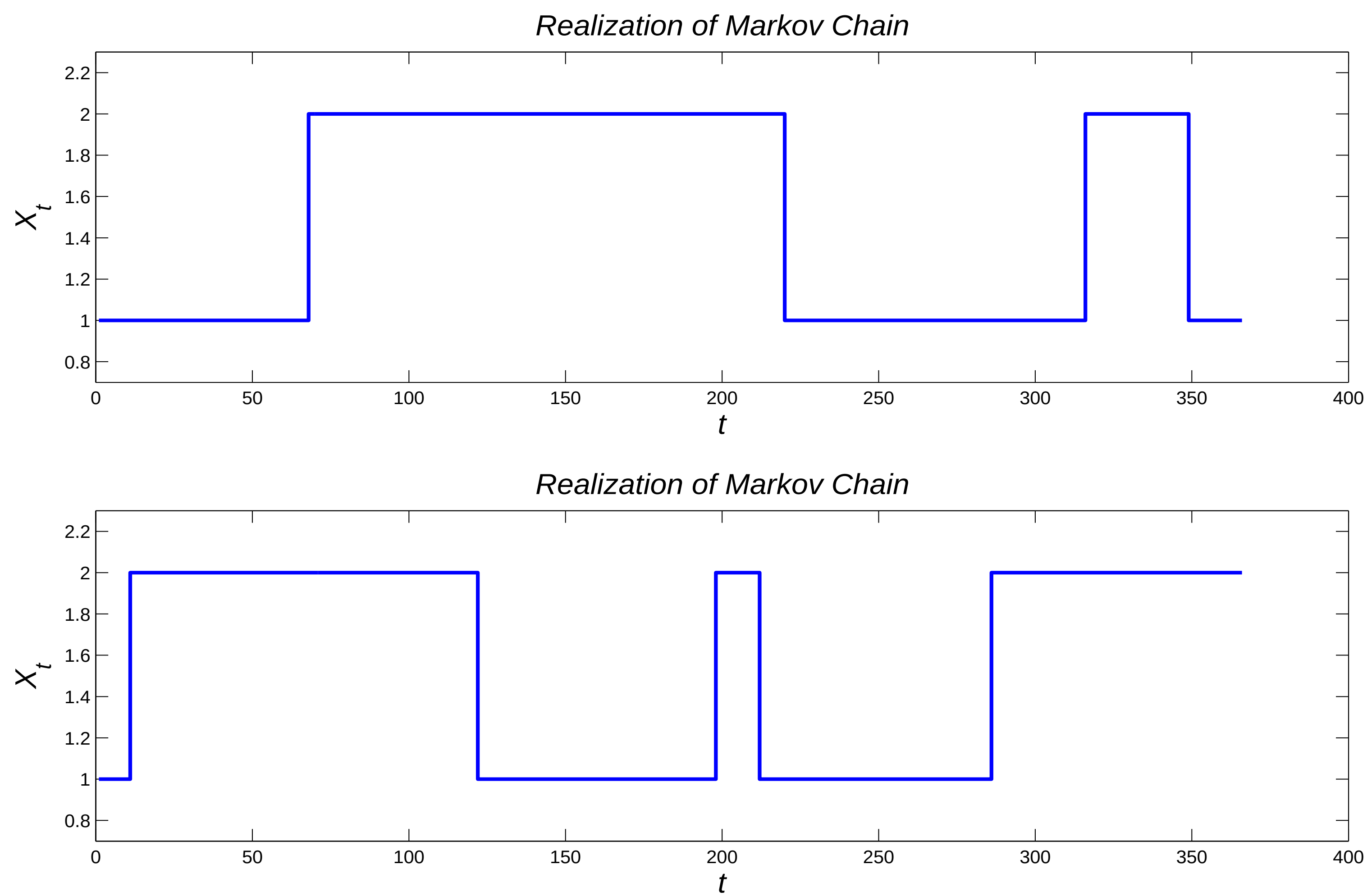

Figure 20: Different realizations of the Markov Chains.

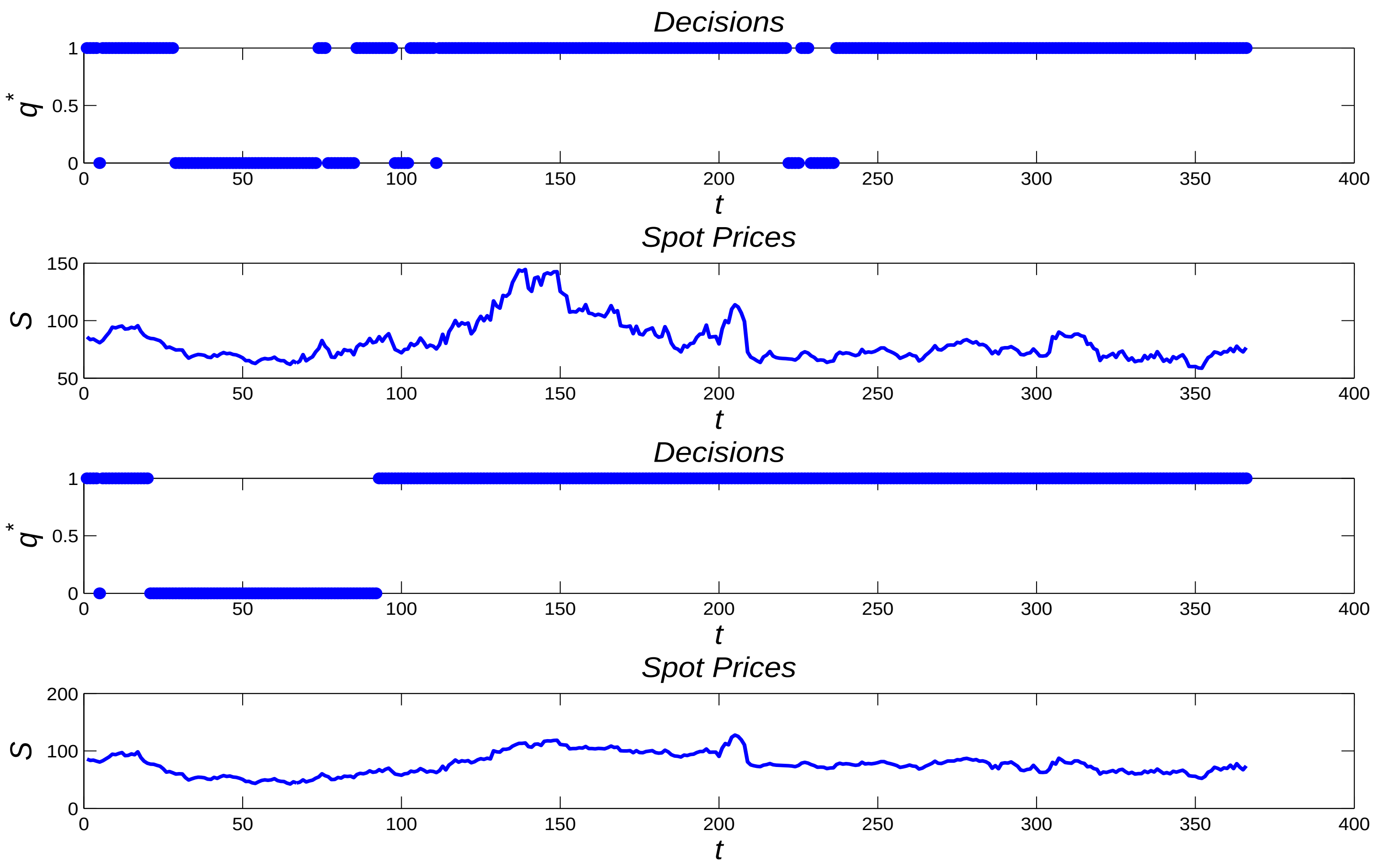

Figure 21: Different decisions and the spot price evolutions.

8 Conclusions

- Set up swing option contracts
- Allowed for make-up and carry-forward banks
- Regime Switching model for forward curve dynamics
- Implement the pentanomial tree approach
- Some numerical examples
- Future work
  - Hedging strategies.

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