Quantum Optimization Workshop Abstracts

October 27-29, 2014

http://www.fields.utoronto.ca/programs/scientific/14-15/quantumopt/

Fields Institute, 222 College St. Toronto

Organizing Committee:

Thomas F. Coleman, Ilias S. Kotsireas, Michele Mosca, Panos M. Pardalos, Rolando Somma

1 PLENARY TALKS

Author: Sergio Boixo
Affiliation: Google
Title: Introduction to quantum annealing
Abstract: I will give an introduction to quantum annealing which assumes no previous knowledge of quantum mechanics. Simulated annealing is a well known discrete optimization algorithm which relies on simulated thermal fluctuations. Quantum annealing is a related method which employs quantum fluctuations. I will review some known similarities and differences between these two methods. For instance, it is theoretically possible to obtain a quadratic speedup with quantum annealing for any given simulated annealing algorithm. Using more general methods (diabatic transitions) it is possible to prove for some problems an exponential speedup (in the oracle model) over the best classical algorithms. Finally, I will present some recent experimental evidence for computational quantum tunneling.

Author: Richard Cleve
Affiliation: University of Waterloo
Title: Two basic quantum paradigms: eigenvalue estimation and amplitude amplification
Abstract: We begin by briefly reviewing some basics of the quantum information framework, such as the notion of a quantum state and the operations that can be performed on it. Then we focus on two basic algorithmic techniques: eigenvalue estimation (used in Shor's factoring algorithm) and amplitude amplification (used in Grover's search algorithm).

Author: Andy Conn
Affiliation: IBM
Title: Some Challenging Practical Problems in Optimization
Abstract: I will discuss problems I have been involved in at IBM. The aim will be to indicate some of the issues that arise in practice that we tend to not pay much attention to in theory. I will also demonstrate

how theoretical results can be motivated by indirect practical issues, including some of "our" failures that I personally find frustrating and some of my prejudices that the audience might find equally frustrating

---

AUTHOR: Robin Kothari
AFFILIATION: MIT
TITLE: Quantum algorithms for simulating physical systems and solving linear systems of equations
ABSTRACT: Simulating physical systems is a major potential application of quantum computers. Today, a significant fraction of the world's computing power is spent in simulating quantum systems that arise in chemistry, materials science, physics, etc. I will survey known quantum algorithms for this task and describe some applications. These algorithms can also be used to solve problems that seem unrelated to simulation, such as solving linear systems of equations. Quantum computers are able to solve a problem related to solving linear systems of equations exponentially faster than any known classical algorithm. I will survey this result and related works.

AUTHOR: Ashwin Nayak
AFFILIATION: Institute for Quantum Computing, University of Waterloo
TITLE: Span programs and other quantum algorithms tools
ABSTRACT: Through this talk, we will learn about span programs and their use in quantum algorithms. Span programs reduce the task of algorithm design to the construction of an abstract linear algebraic object. They also compose readily, without the usual loss in efficiency in composing bounded error algorithms. Remarkably, the best span programs correspond to quantum algorithms with the best query complexity. This illustrates an elegant application of semi-definite programming duality in quantum computation.

AUTHOR: Panos M. Pardalos
AFFILIATION: University of Florida
TITLE: Computational Models and Hard Optimization Problems
ABSTRACT: Most of the conventional computer models are based on the von Neumann computer architecture and the Turing machine model. However, quantum computers (several versions!), analog computers, dna computers, and several other exotic models have been proposed in an attempt to deal with intractable problems.

We are going to give a brief overview of different computing models and discuss several classes of optimization problems that remain very difficult to solve. Such problems include graph problems, nonlinear assignment problems, and global optimization problems.

AUTHOR: William Pulleyblank
AFFILIATION: USMA
TITLE: Linear and Integer Programming and Combinatorial Optimization
ABSTRACT: Historically, the development of algorithms has been highly influenced by current computer architectures. We discuss several paradigms used in integer and linear programming as well as specialized combinatorial optimization methods developed for certain classes of problems. These have often exploited the most powerful computers available. Hopefully, some will be candidates for quantum computing.

Author: Krysta Svore
Affiliation: Microsoft
Title: Quantum Computing: Transforming the Digital Age
Abstract: In 1981, Richard Feynman proposed a device called a "quantum computer" to take advantage of the laws of quantum physics to achieve computational speed-ups over classical methods. Quantum computing promises to revolutionize how we compute. Over the course of three decades, quantum algorithms have been developed that offer fast solutions to problems in a variety of fields including number theory, optimization, chemistry, physics, and materials science. Quantum devices have also significantly advanced such that components of a scalable quantum computer have been demonstrated in a variety of quantum systems. In this talk, I will attempt to reveal some of the mysteries of this disruptive computational paradigm. I will showcase recent advances in quantum algorithms for real-world applications and in scalable, fault-tolerant devices.

Author: Mario Szegedy
Affiliation: Rutgers University
Title: Searching by quantum walks
Abstract: Spatial search by quantum walk is database search with the additional constraint that one must move through the search space via a quantum walk that obeys some locality structure (grid, hypercube, etc.). Quantum algorithms based on spatial search frequently outperform their classical counterparts, allowing one to achieve as much as a quadratic speed-up. The realization of this speed-up however often needs resourceful thinking, as we shall demonstrate using examples ranging from triangle finding to matrix multiplication.

---

2 CONTRIBUTED TALKS

Author: Srinivasan Arunachalam
Affiliation: Centrum Wiskunde & Informatica (CWI), Netherlands
Title: Quantum Speed-ups for Optimization problems
Abstract: In this talk, we consider two important optimization problems, Boolean satisfiability (SAT) and Derivative-free optimization (DFO). We compare quantum algorithmic speed-ups for these problems with the state-of-the-art classical algorithms. Often, with quantum algorithmic improvements proposed in the literature, it is generally assumed that the speed-up follows in a straightforward manner for industrially relevant problems relying on these algorithms. We consider two important practical problems in the areas of computational bio-informatics and civil engineering which rely on solving SAT and DFO, respectively. We investigate if known quantum techniques to speed-up these algorithms directly benefit the industries interested in solving these problems. We highlight some immediate hurdles, both from an algorithmic and from an architecture perspective, in obtaining a convincing quantum speed-up for practical problems.

This talk will be based on my Master's thesis "Quantum Speed-ups for Boolean Satisfiability and Derivative-Free Optimization", which was supervised by Michele Mosca.

Author: Michael Jarret
Affiliation: University of Maryland
Title: Adiabatic Optimization and Dirichlet Graph Spectra
Abstract: Several previous works have investigated the circumstances under which quantum adiabatic optimization algorithms can tunnel out of local energy minima that trap simulated annealing or other classical local search algorithms. Here we pursue two particular questions: (1) do adiabatic optimization algorithms always succeed in polynomial time for trivial optimization problems in which there is only one local minimum and (2) what characterizes the boundary between large- and small- gapped Hamiltonians? In addressing the first, we surprisingly find a counterexample in which the potential is a single basin on a graph, but the eigenvalue gap is exponentially small as a function of the number of vertices. For the second, we discover an equivalence between adiabatic interpolation Hamiltonians and the problem of determining the spectral gap of the Dirichlet spectrum of a weighted graph. From this observation, we derive a number of bounds on the eigenvalue gap based on well-studied graph quantities. Additionally, in the context of quantum systems exhibiting path- and hypercube-like connectivity, we obtain a discrete analogue to the "fundamental gap theorem," yielding a tight lower bound to the eigenvalue gap in the presence of convex potentials.

Coauthors: Stephen P. Jordan

Author: Stacey Jeffery
Affiliation: Caltech
Title: Parallel quantum algorithms
Abstract: In classical algorithms, parallel computing is of interest, because for many problems, we can solve the problem significantly faster by devoting more computational resources to the problem. In order to compare the relative power of classical and quantum computers, for these problems, we need to consider the best parallel quantum algorithms, and how they compare to the best parallel classical algorithms.

In this talk, I will briefly outline results and open problems in parallel quantum algorithms, and discuss how quantum algorithmic techniques can be applied in the parallel setting.

Author: Catherine McGeoch
Affiliation: D-Wave Systems / Amherst College
Title: Toward a Benchmark Test Suite for Quantum Annealers
Abstract: D-Wave quantum annealing system bundles a heuristic optimization algorithm together with a dedicated quantum platform. Empirical evaluation of this system must draw on methodological standards developed in heretofore distinct research areas, including experimental study of heuristics for NP-Hard problems, and computer performance analysis and HPC. Furthermore, the analog and quantum nature of these novel systems introduces whole new sets of questions about best practice in experimental evaluation. How do we compare the time of a single analog computation to that of a classical instruction block? How do we account for effects of the open system when building a performance model? I will discuss these challenges in the context of a concrete and immediate problem: selecting a suite of instance classes suitable for evaluating performance of D-Wave systems versus classical optimization solvers. Is

not enough to consider abstract problem complexity; one must also take into account the quantum performance model, specific features of the quantum annealing algorithm, and limitations imposed by the analog hardware.

Author: Ojas Parekh
Affiliation: Sandia National Labs
Title: Unearthing planted solutions in quantum-annealing-inspired Ising problems
Abstract: In order to assess the potential of emerging quantum computers, such as D-Wave System's quantum annealers, we seek hard instances of interesting combinatorial optimization problems. Although computational results may hint at hardness, inferring formal worst-case hardness from practical or randomly generated instances is difficult. We consider two classes of (classical) Ising spin glass instances recently considered in the context of benchmarking D-Wave's quantum annealers.

The first class, restricted to edge weights of -1 and +1 with no linear term, have been the primary focus of seminal D-Wave benchmarking studies. Although these instances were assumed to be NP-hard based on the classical work of Barahona, Barahona's proofs do not apply to them. We show that these instances are indeed NP-hard.

The second class, which was recently presented by Itay Hen in the 2014 Workshop on Adiabatic Quantum Computing, extends the idea of planted optimal solutions to Ising instances. Planted solutions are important for benchmarking since an optimal solution is not always provably and readily attainable. Some of the planted-solution instances of Hen and his collaborators are extremely hard for the D-Wave Two annealer, while D-Wave outperforms select classical heuristics on others. In fact it appears that this is one of the only class of instances known for which D-Wave appears to perform best; however, empirical results still seem to indicate super-polynomial scaling in run time.

We show that the optimal solution value of all of the above planted-solution instances is attainable in (classical) polynomial time via Barahona and Mahjoub's metric polytope relaxation for the maximum cut (and Ising) problems. We note, however, we cannot directly obtain an optimal solution with this approach, since the polytope is not necessarily integral on these instances. This touches on the fundamental question of whether a polynomial-time algorithm for computing the optimal value of an optimization problem implies a polynomial-time algorithm for actually generating an optimal solution. For many fundamental combinatorial optimization problems, self-reducibility provides an affirmative answer; however, self-reducibility is more difficult to leverage in the case of structurally restricted instances. We discuss these broader issues in the context of the planted-solution Ising instances.

Coauthors: Robert D. Carr (Sandia National Labs)

Author: Pooya Ronagh
Affiliation: 1QB Information Technologies
Title: A Branch and Bound Method Based on Lagrangian Relaxation and Quantum Annealing
Abstract: Quantum annealing is perceived useful for binary quadratic programming (BQP) problems that are a priori unconstrained. In the case of constrained problems, it is a common practice to relax linear equality constraints as penalty terms in the objective function. However, there has not been a proposed method for efficiently dealing with inequality constraints yet. In this talk, we will explain a method for solving Lagrangian dual of a BQP problem in presence of inequality constraints using D-Wave

systems and will employ this procedure in a branch and bound framework for general BQP problems.

Coauthors: Ehsan Iranmanesh, Brad Woods

Author: Henry Wolkowicz
Affiliation: University of Waterloo
Title: Alternating projection methods and quantum information science
Abstract: We consider the problem of constructing quantum operations or channels transforming a given set of quantum states. In the mathematical setting, the problem reduces to finding a completely positive linear map, if it exists, that maps a given set of density matrices to another given set of density matrices. This problem in turns is equivalent to constructing a positive semidefinite matrix satisfying certain linear constraints. We show that the problem is never "weakly infeasible". Furthermoren, we exploit the structure of the problem and develop efficient methods in solving instances of high dimensions that standard linear semidefinite programming solvers cannot handle. We present numerical experiments based on various "alternating projection type" methods.

(work with Vris Yuen-Lam Cheung, Dmitriy Drusvyatskiy, Chi-Kwong Li, Diane Pelejo)

---

3 POSTERS

Author: William de la Cruz de los Santos
Affiliation: Universidad Autonoma, Mexico
Title: Solving NP-hard combinatorial optimization problems with adiabatic quantum computing
Abstract: A construction of a Hamiltonian path that allows the treatment of quadratic pseudo-Boolean optimization with Adiabatic Quantum Computing is introduced. We focus on Hamiltonians realizable as two-local interaction Ising systems that appear in a natural correspondence among optimization Boolean variables and quantum bits. Combinatorial graph problems are in correspondence with pseudo-Boolean maps that in turn are reduced in polynomial time to quadratic maps. In particular, any NP-hard optimization problem can be solved by reducing it to the Maximum Independent Set problem, then through the equivalent formulation of quadratic Boolean maps optimization by slowly evolving the corresponding the quantum system in an adiabatic processing.

Author: Joshua Job
Affiliation: University of Southern California
Title: Planted solutions and frustrated loops on the D-Wave Two
Abstract: We present a new method to construct a set of Ising-type optimization problems with tunable hardness. The problems are generated around predetermined ground-state configurations, called planted solutions, which makes them suitable for benchmarking purposes and which allows for some control over the frustration of their solutions, and in turn over their hardness. The method borrows ideas from the realm of constraint satisfaction (SAT), and in turn the problem set exhibits similar properties of planted SAT problems, such as a phase transition from a many-solution phase to a one-solution (only the planted

solution) phase, with the phase transition signifying a peak in the typical hardness of the problems. We use the problems to compare the performance of the D-Wave Two chip against several classical solvers.

Coauthors: Itay Hen, Tameem Albash, Troels Rnnow, Matthias Troyer, Daniel Lidar

---

AUTHOR: Bill Kaminsky
AFFILIATION: MIT
TITLE: Assessing the Plausibility and Practicality of Adiabatic Quantum Computation Efficiently Producing *Approximate* Solutions to Ising Model Ground State Problems at Accuracies for which All Known Classical Algorithms Are Inefficient
ABSTRACT: In recent years, several research groups have presented substantial evidence that adiabatic quantum computation based upon Ising models subjected to a homogeneous transverse field (henceforth, "standard AQC") cannot produce *exact* solutions to interesting minimization problems exponentially faster than known classical algorithms. This inability appears to hold already in the ideal case of zero temperature, let alone in the practical case of a nonzero temperature that cannot shrink as the problem size grows. However, the following question remains open: *when standard AQC tackles practically interesting minimization problems, can it produce approximate solutions in polynomial time with such high accuracy that all known classical minimization algorithms would take exponential time?* In this poster, we present a theoretical argument that the answer to this question not only could be yes at zero temperature, but also it could remain yes even at a nonzero temperature that is independent of the problem size.

Our argument has three parts. First, in the case of zero temperature, we explain how changing the goal of standard AQC from exact solution to approximation drastically changes the necessary condition for standard AQC to be efficient. Briefly summarizing, exact solution tolerates no significant transitions whatsoever of probability amplitude from the AQC's ground state to any of its excited states. In sharp contrast, nontrivial approximation can tolerate even exponentially many transitions of probability amplitude between pairs of ever-higher excited states that furthermore might even be separated by exponentially small energy gaps. This wide tolerance arises because there are many interesting Ising models on N spins for which producing even the $O(2^{\alpha N})$th excited state (where $0 < \alpha \leq 0.2$ is an N-independent constant) is a task that makes all classical stochastic local search algorithms take exponential time.

Second, we assess how plausible it is that such an exponentially-large-yet-vanishingly-small-relative-to-2^N number of energy level crossings occurs in standard AQC restricted to run quickly in just $O[\text{Poly}(N)]$ time. To this end, we demonstrate how perturbation theory around the initial transverse-field-only Hamiltonian of standard AQC combined with Hermite-Padé interpolation each potential avoided energy level crossing can provide an upper bound on the energy that obtains after all the crossings that shall occur over the course of the AQC.

Third and finally, we make the case that using standard AQC for approximation is quite robust against the practical imposition of an operating temperature that cannot shrink as the problem size grows. Specifically, we develop a moment-based method to produce upper bounds on the number of excited states within $O(1)$ of the ground state energy of not just a standard AQC Hamiltonian, but in fact within $O(1)$ of the ground state energy of any N-qubit, $[k = O(1)]$-local, QMA-complete Hamiltonian $X + Z$ where X is diagonal in the Pauli-x product basis and Z is diagonal in the Pauli-z product basis. With this method, we prove that *given the promise that both summands X and Z have close-to-Gaussian eigenenergy distributions*, the number of excited states within $O(1)$ of the ground state energy is at most

subexponential — that is, $O[\exp(N^{1/2+\epsilon})]$, where $\epsilon > 0$ can be as small as desired. We close by discussing how this subexponential upper bound might be strengthened to a $O[\mathrm{Poly}(N)]$ upper bound.

Author: Maria Kieferova
Affiliation: IQC, University of Waterloo
Title: On The Power Of Coherently Controlled Quantum Adiabatic Evolutions
Abstract: We provide a new approach to adiabatic state preparation that uses coherent control and measurement to average different adiabatic evolutions in ways that cause their diabatic errors to cancel, allowing highly accurate state preparations using less time than conventional approaches. We show this new model for adiabatic state preparation is polynomially equivalent to conventional adiabatic quantum computation by providing upper bounds on the cost of simulating such evolutions on a circuit based quantum computer. Finally, we show that this approach is robust to small errors in the quantum control register and that the system remains protected against noise on the adiabatic register by the spectral gap.

Coauthors: Nathan Wiebe

`arXiv:1403.6545`

Author: Shelby Kimmel
Affiliation: University of Maryland
Title: Oracles with Costs
Abstract: While powerful tools have been developed to analyze quantum query complexity, there are still many natural problems that do not fit neatly into the black box model of oracles, including many optimization problems. We create a new model that allows multiple oracles with differing costs. This model captures more of the difficulty of certain natural problems. We test this model on a simple problem, Search with Two Oracles, for which we create a quantum algorithm that we prove to be asymptotically optimal. We further give some evidence, using the geometric picture of Grover's algorithm, that no algorithm can have less cost than ours by even a constant multiplicative factor.

Coauthors: Cedric Lin, Han-Hsuan Lin

Author: Andrew D. King
Affiliation: D-Wave Systems
Title: Quantum optimization and constraint satisfaction with perturbed Hamiltonians
Abstract: Work from several groups of researchers has shown promise in solving constraint satisfaction problems on the D-Wave hardware, which implements open-system quantum annealing. These problems include random Boolean satisfiability instances with MAX-CUT clauses near the phase transition, and conjunctions of frustrated cycles on an arbitrary hardware graph. Of crucial importance is the issue of mitigating the effect of perturbations in the Hamiltonian that result from hardware imperfection. In this talk I will discuss the general framework for constraint satisfaction in the Ising model, recent advances in the state of the art, and how we might optimize performance in this arena in the near term.

Author: Ilias S. Kotsireas
Affiliation: Wilfrid Laurier University
Title: D-optimal matrices
Abstract: D-optimal matrices are square matrices of even order with elements from $\{-1, +1\}$ that have maximal determinant. Finding D-optimal matrices of various orders is a very challenging and hard combinatorial problem. One of the most fertile and successful methods to find D-optimal matrices has been to look for a special kind called circulant D-optimal matrices. We will present circulant D-optimal matrices and their properties in detail. We will also present algorithms used to find circulant D-optimal matrices.

Author: Davide Venturelli
Affiliation: U.S.R.A. Quantum AI Lab at NASA ARC
Title: Quantum Optimization of Fully-Connected Spin Glasses
Abstract: The Sherrington-Kirkpatrick model with random 1 couplings is programmed on the D-Wave Two annealer featuring 509 qubits interacting on a Chimera-type graph. The performance of the optimizer compares and correlates to simulated annealing. When considering the effect of the static noise, which degrades the performance of the annealer, one can estimate an improvement on the comparative scaling of the two methods in favor of the D-Wave machine. The optimal choice of parameters of the embedding on the Chimera graph is shown to be associated to the emergence of the spin-glass critical temperature of the embedded problem.

Coauthors: Salvatore Mandr, Sergey Knysh, Bryan O'Gorman, Rupak Biswas, Vadim Smelyanskiy

---